Effects of the TOUCHMATH Program Compared to a Number Line Strategy to Teach Addition Facts to Middle School Students with Moderate Intellectual Disabilities

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Abstract: The purpose of this study was to systematically replicate and extend previous studies of the TOUCHMATH program, a multi-sensory mathematics program (Bullock, Pierce, & McClellan, 1989). Three middle school students with moderate and multiple disabilities (e.g., autism and moderate intellectual disabilities) participated. Students were taught how to solve single-digit mathematics problems using TOUCHMATH and a number line. An alternating-treatments design across participants (Barlow & Hersen, 1984) was utilized to evaluate and compare the effects of both strategies. Results indicated that the TOUCHMATH strategy was more effective and efficient in teaching students’ single-digit addition problems compared to the use of the number line. Limitations of the study, implications for practice for classroom teachers, and suggestions for future research are discussed.

Many students with disabilities at the middle school level, particularly those with moderate intellectual disabilities have difficulty meeting the curriculum demands in content-area classrooms such as mathematics instruction (see Browder & Grasso, 1999; Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008; Butler, Miller, Kit-hung, & Pierce, 2001; Jitendra & Xin, 1997; Kroesbergen & Van Luit, 2003; Mastropieri, Bakken, & Scruggs, 1991; Miller, Butler, & Lee, 1998; Swanson & Jerman, 2006, Xin & Jitendra, 1999; for reviews). Specifically, students with moderate intellectual disabilities frequently have difficulties with mathematics, including basic skills (Nesbitt-Vacc & Cannon, 1991; Podell, Tournaki-Rein, & Lin, 1992; Luit & Naglieri, 1999; Young, Baker, & Martin, 1990), money applications (Test, Howell, Burkhart, & Beroth, 1993; Fredrick-Dugan, Test, & Varn, 1991; Sandknop, Schuster, Wolery, & Cross, 1992), and problem-solving activities (Mastropieri, Scruggs, & Shiah, 1997; Morin & Miller, 1998). For example, students with moderate intellectual disabilities are less proficient and use less effective strategy instruction in completing and solving mathematics problems than their “typically” functioning peers (Goldman, Pellegrino, & Mertz, 1988). However, performing basic computational mathematics is essential for student success and to foster independent living skills. Acquiring these computational skills for many students with moderate intellectual disabilities may require the use of manipulatives.

Using manipulative materials has been used to assist in teaching basic computational mathematics skills for students with moderate intellectual disabilities. For instance, Burns (1996) indicated that manipulative materials were used at all levels and that teachers could not teach without them. There are various manipulatives that are used in teaching basic computational mathematics skills. For example, one widely used mathematics strategy to teach mathematics is the number line (Ernest, 1985). Copeland, Hughes, Agran, Wehmeyer, and Fowler (2002) used a number line and

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systematic instruction to teach four students with moderate intellectual disabilities to match numbers at the secondary level. The use of a number line is one multi-sensory approach to teach addition utilizing a count-all to count-on approach (Secada, Fuson, & Hall, 1983). In counting-all, entities must be present for each addend, and students count all the entities. For instance, students begin counting “one” and count to “m + n.” Where as counting-on, the student begins with “m” and continues counting on “m + n” (Secada et al., 1983). For example, if a student was given the following problem 4 + 5, the student would start counting-on from four and say, “five, six, seven, eight, and nine.”

Another type of manipulative strategy for teaching computational mathematics skills is the use of a “dot-notation” as a representative approach. Kramer and Krug (1973) introduced the dot-notation approach for teaching mathematics skills to students with disabilities, which students begin learning to count-all, and progress to using the counting-on approach. Kokaska (1975) examined the effectiveness of the dot notation system with four students with mild disabilities. These students were pre-tested in their addition and subtraction skills. Two of the students were unable to independently complete any problems, while the other two students were only able to complete a partial number of the problems. Then, these students were taught the dot-notation system for solving addition and subtraction problems. Results showed that two of the four students were able to solve addition problems ranging from single-digits to a combination of two or more digits. Moreover, students solved problems accurately if the problems were presented either horizontally or vertically. These results support the use of multi-sensory manipulative programs such as the TOUCHMATH program.

The TOUCHMATH Program (Bullock et al., 1989) which is similar to that of the dot-notation introduced by Kramer and Krug (1973) was used with large numbers of students in the general education classroom. Scott (1993) examined the use of the TOUCHMATH program with elementary students with mild disabilities in the fourth grade. Three students with mild disabilities were taught the touch-points strategy for addition and subtraction problems within a multiple-probe design across mathematics skills. The target skills were adding two-digit numbers with regrouping, adding columns of two-digit numbers with regrouping, and subtracting single-digits up to 18, subtracting two-digit numbers with regrouping, and subtracting three digit numbers with regrouping. Results indicated that all three participants were successful in using the TOUCHMATH program. In another study, Simon and Hanrahan (2004) utilized the TOUCHMATH program to evaluate the learning of addition computational skills with students with learning disabilities. Three students with learning disabilities in mathematics instruction were examined to see if they could be taught three-row, double-digit addition problems using the dot-notation method. Results revealed that the students were able to learn and apply the dot-notation method and were able to retain the method from 6 weeks and 18 weeks after completing instruction.

With different manipulative materials available, the suitability of a particular manipulative is a concern, especially if a student is dependent upon a manipulative system to complete computational problems later in life (Kramer et al., 1973). To date, there are a limited number of studies involving students with moderate and multiple disabilities using a dot-notation method. Moreover, previous studies have only included elementary-aged students and no studies using the dot-notation strategy in the middle school grades were found.

Therefore, the purpose of this study was to replicate and extend on the use of the “touch point” strategy to teach addition problem-solving skills to students with moderate intellectual disabilities at the middle school level. In addition, a secondary purpose of this study was to compare the effectiveness and efficiency of touch points and number lines for teaching single-digit addition problems to determine if there are functional differences between the two strategies for students with moderate intellectual disabilities (e.g., autism and moderate intellectual disabilities).
Method

Participants and Setting

The sample consisted of three middle school students with moderate intellectual disabilities. Two of the students were diagnosed with autism in addition to having a moderate intellectual disability. Formal educational assessments were conducted on these two students, particularly with autism, as well as the local educational agency’s school psychologist within four years preceding the study. The teacher participating in the study had three years of teaching experience in a self-contained special education classroom working with students with mild to moderate intellectual disabilities, while the paraprofessional had four years experience in an early education classroom setting and one year in a self-contained special education classroom environment. The study was conducted within a self-contained classroom, which instruction commonly occurred for these students. All the students demonstrated the prerequisite skills of counting and writing to 20.

Ashley. Ashley was a 13-year-old female in the sixth grade that on the Wechsler Intelligence Scale for Children – WISC IV (Wechsler, 1991) had a Full Scale IQ score of 40 and was diagnosed with a moderate intellectual disability. Ashley’s Verbal Comprehension and Perceptual Reasoning were also below average, as her standard scores were 53 and 47, respectively. Also, her Working Memory and Processing Speed were likewise far below average with a standard score of 50. According to her Peabody Picture Vocabulary Test-4 scores, her receptive language skills were also very low with a standard score of 50; similarly Ashley’s expressive language skills on the Expressive One-Word Picture Vocabulary Test were below average with a standard score of 48. In addition, Ashley’s basic academic skills were also well below average, specifically her phonemic decoding skills were below average on the WJ-III Word Attack subtest with a standard score of 31, word identification skills on the WJ-III Letter-Word Identification Test with a standard score of 37, and finally on the WJ-III Spelling and Calculation subtests, with standards scores of 36 and 7, respectively, which were both well below average. Furthermore, her scores from the Vineland Adaptive Behavior Scale were also in the low range as her standard scores of adaptive skills were as follows: Communication, 54; Daily Living Skills, 52; Socialization, 89; Motor Skills, 49; and Composite, 56. Ashley was not currently taking any medications; however, based on her last IEP, she was functioning approximately in the lower first grade level in mathematics. Moreover, Ashley had a mild articulation disorder and received speech and language services and also displayed deficits in writing (i.e., poor graph-motor control) and had problems with her short-term attention abilities.

Robert. Robert was a 13-year-old male in the seventh grade and according to the Stanford-Binet Intelligence Scale–Fourth Edition (Thorndike, Hagan, & Sattler, 1986) his scores showed the following: Verbal Reasoning, 70; Visual Reasoning, 63; Quantitative Reasoning 66; Short-Term Memory, 49; and Test Composite, 54. As indicated by his Peabody Picture Vocabulary Test-R scores, Robert was within the moderate intellectual disability range. Robert was also administered the Gilliam Autism Rating Scale (Gilliam, 1995) and was diagnosed with autism. The scores from the Gilliam Autism Rating Scale (GARS) are as follows: Stereotyped Behaviors, 8; Communication 9; Social Interaction, 8; Developmental, 9; and Autism Quotient, 85. Additionally, his scores from the Vineland Adaptive Behavior Scale were in the low range, as average scores range from 85–115 and his standard scores of adaptive skills were as follows: Communication, 53; Daily Living Skills, 44; Socialization, 64; and Composite, 50. Prior to beginning the study, Robert could only add single-digit addition problems that involved one as one of the numbers. Robert was also Hearing Impaired and used two hearing aides and was not currently taking any medications.

Ken. Ken was a 14-year-old male in the eighth grade and had a Full Scale IQ score of 50 on the Wechsler Intelligence Scale for Children – WISC III (Wechsler, 1991), which suggested he had a moderate intellectual disability. Ken’s Verbal Comprehension and Perceptual Reasoning were also below average, as his standard scores were 55 and 53, respectively. Also, his Working Memory and Processing Speed were likewise far below average with standard scores of 56 and 53, respectively. Similar to Ashley, according to Ken’s Peabody Picture Vocabulary Test-4 scores, his receptive language skills were
also extremely low with a standard score of 60. Ken was also administered the *Childhood Autism Rating Scale* (Schopler, Reichler, & Renner, 1988) and was diagnosed with moderate autism, with a standard score on the (CARS) of 31. In addition, his scores from the *Vineland Adaptive Behavior Scale* were in the low range, as his standard scores of adaptive skills were as follows: *Communication, 72; Daily Living Skills, 65; Socialization, 66; and Composite, 66.* Prior to the study, Ken was unable to add single-digit addition problems independently during mathematics instruction. Lastly, Ken was taking Stratera 40mg once a day for his attention deficit disorder (ADD).

**Materials**

The materials used in this study consisted of two forms, Form A and Form B, of which each form contained 10 single-digit addition mathematics problems. Form A consisted of the *TOUCHMATH* (Bullock et al., 1989) single-digit addition problems worksheets with the “touch points” presented on the numbers, while Form B contained the opposite problems and in a different order on the page for use with the number line strategy. For instance; if Form A had the problem $5 + 8$, then Form B presented the problem as $8 + 5$ located in a different order of the worksheet. A number line with numbers from 0 to 20 also was used.

**Dependent and Independent Variables**

The dependent variable was the percentage of single-digit mathematics problems performed correctly by the three students, while the independent variables were the use of the *TOUCHMATH* program using “touch points” and the number line strategy. There were two mathematics worksheets with different single-digit mathematics problems of similar difficulty. Also, the worksheets (both Form A and B) presented single-digit addition problems presented vertically with comparable problems for the students to solve. For example, if worksheet Form A had $3 + 5$ then worksheet, Form B had $5 + 3$ except in a different order on the page that was semi-randomly assigned. Every effort was made to ensure that the problems were not situated in the same order or placement on the sheets. While similar problems were presented on both Forms A and B, the same worksheet was never presented to the students twice during the intervention phases.

**Data Collection and Experimental Research Design**

The data collection procedures for each session consisted of the permanent product recording from each of the three students. In order to calculate the student’s percentage of correct mathematics problems, each of the sessions were calculated by taking the number of correct mathematics problems completed independently and dividing by the total number of problems presented to calculate the total number of problems completed correctly for each of the sessions. The criterion for acquisition of the student’s performance of the mathematics problem-solving was 100% correct for three consecutive sessions. The strategy to first achieve criterion was then replicated using the content from the strategy that did not meet criterion. An alternating-treatments design (Barlow & Hersen, 1984) was used to examine and compare the differential effects of the *TOUCHMATH* program using “touch points” and the number line strategy on the acquisition of mathematics performance for each of the three students.

**Procedure**

**Baseline.** In the baseline phase, the students were provided a worksheet with 10 single-digit addition mathematics problems to complete. During this phase, the probe was completed when the students answered all 10 of the mathematics problems or if there were no written response on the worksheet after a period of 15 minutes from the student. In this phase, the students also did not receive any additional teacher assistance, as they completed the worksheet. In addition, before proceeding to the intervention phases, a minimum of three consecutive sessions of stable data collection were required for all three students.

**Intervention procedures.** During both interventions, the *TOUCHMATH* program using “touch points” and the use of the number line
strategy, the students were seated at their desks with the instructor in their classroom. An adapted model-lead-test procedure (Adams & Engelmann, 1996) was utilized to teach the students how to use the “touch points” and number line strategy. In this procedure, the teacher modeled the correct responses, lead the students by having them state the correct responses with the teacher, and then tested them by having the student independently state the correct responses. In the first three sessions following the baseline phase, the teacher provided model training sessions. During the “touch point” instruction, the students were explicitly and directly instructed on the dot-notation positions of the numbers 1 through 9, using the TOUCHMATH program. Then, the teacher provided the students with one worksheet per session of 10 single-digit addition problems with illustrated “touch points”, which consisted of numbers with single or double black dots on the worksheets. The students were instructed to count aloud the number of dots, while touching each dot, of both numbers (count-all) and then write the last number they had stated.

During the number line strategy instruction condition, the teacher provided the students with a number line from 0 to 20. Students were asked to place their fingers on the number line at the number that matches the first number in the problem; then locate the other number and move that many spaces counting aloud (count-on); and then write the number. In this condition, the teacher praised the student orally for the correct response followed by an immediate imitation (lead) of the model, which was identical to the “touch points” phase. Finally, the remaining sessions for both the “touch points” and number line strategy were considered test sessions to be recorded by the researcher.

The test sessions for both of the strategies “touch points” and the number line intervention were presented once a day in the morning of the school day, with at least a 30 minute but not more than one hour break between each testing session. During the break sessions, the students were instructed to work on other academic content-areas and homework assignments. The presentations of both interventions were administered semi-randomly to counterbalance which strategy was implemented first or second during the testing sessions. Students completed one mathematics worksheet with the 10 single-digit addition mathematics problem per session using either the “touch points” or number line strategy. Sessions ranged from 5 to 15 minutes. In addition, students received verbal praise for correct responses. However, if a student required assistance, a least-to-most prompt hierarchy was used until the students provided a response without assistance using a 10-second interval between each prompt level (a) verbal prompting, (e.g. “Do you see what numbers need to be added? Do you see the number line? Do you see the touch points?”), (b) followed when needed by gesturing (e.g. pointing to the first number on the number line or touch points after the verbal prompting has received a response with no further actions from the student), and (c) modeling or demonstrating (ensuring the students repeated what the teacher said and pointed and stated the number and counted correctly).

**Replication.** During the replication phase, the first strategy to achieve the criterion was then replicated using the content from the nonpreferred strategy. The criterion was answering problems with 100% accuracy for three consecutive sessions.

**Reliability**

The investigator, classroom teacher, and the paraprofessional in the classroom collected data for inter-observer reliability and procedural reliability measures. Inter-observer and procedural reliability data was collected during the baseline, intervention, and replication phases. The observers independently and simultaneously recorded the number of single-digit addition mathematics problems scored correctly, and the required prompt level. The inter-observer agreement was calculated by dividing the number of agreements of student responses by the number of agreements plus disagreements and multiplying by 100 for 25% of the sessions during each of the phases. Inter-observer reliability ranged from 99% to 100% agreement across all three phases. The inter-observer reliability agreement for each student across baseline, interventions, and replication phases was 99% for Ashley, 99% for Robert, and 100% for Ken.
Procedural reliability probes measured the teacher’s performances of implementing the correct mathematical strategy, responding to correct and incorrect responses, prompting hierarchy, and response time. The investigator modeled both intervention strategies (e.g., “touch points” and number line) and prompting hierarchy to both the teacher and para-professional using a checklist of specified teacher behaviors. Upon completion of three consecutive trials with 100% accuracy, the teacher was considered to have mastered the procedures. Procedural reliability probes were conducted in 25% of sessions in each phase of this study. The procedural agreement level was calculated by dividing the number of observed teacher behaviors by the number of planned teacher behaviors and multiplying that by 100. Procedural reliability ranged from 98% to 100%, with a mean of 99%.

Results

As illustrated in Figure 1, all three of the students across baseline, interventions, and replication phases of the intervention showed significant improvements using the “touch points” method compared to the number line strategy to solve single-digit addition mathematics problems. Moreover, the results indicated that all three of the students were able to utilize the “touch point” strategy faster and more accurately than the number line intervention. During the baseline phase, the students averaged 4% of the single-digit mathematics problems accurately, however, while in the “touch points” phase the students averaged 92% of the problems correctly, compared to only 30% while using the number line strategy. Furthermore, all three of the students averaged 96% correct during the replication phase.

Ashley. During the baseline phase, Ashley was only able to complete one single-digit mathematical problem correctly. Although she improved with the “touch point” strategy faster, her data indicated an ascending pattern with both strategies, but was able to reach criterion sooner with the “touch point” strategy. In fact, she did not reach criterion until session 28 even though she was able to achieve 90% accuracy by the fourth session, she could not maintain that accuracy over three sessions until the 28th session. In the replication phase, she dropped to a 90% in the second session of the replication phase, but then she was able to maintain 100% accuracy. Ashley was observed using the “touch point” method during four of the number line strategy sessions. In sessions 9, 16, 20 and 23, Ashley had a peak in her score with using the number line strategy, and a dip of 30% in the “touch point” strategy when she was observed carrying over one strategy to another. However, the use of the touch points was determined to be more effective and efficient based upon Ashley reaching criterion quicker than using the number line. During the replication, the content used during the number line strategy was presented to Ashley. Using the touch-point strategy, Ashley’s single-digit addition performance improved to a mean of 98%.

Robert. Robert was unable to complete any of the single-digit addition problems during the baseline phase. However, he was able to attain 100% accuracy by the 8th and 9th sessions for both strategies, but could not maintain that accuracy to achieve criterion until the 25th session using the “touch point” strategy. In the last three sessions in the number line phase, Robert’s performance was observed descending following a relatively inconsistent performance for using the number line strategy. When replicated, the content used during the number line strategy was presented and Robert’s single-digit addition performance improved to a mean of 100% using the touch-point strategy.

Ken. Ken could not complete with accuracy any single-digit addition problems during the baseline phase. However, Ken was able to obtain 100% accuracy using the “touch point” strategy by the 7th session, but did not achieve criterion until the 17th session. Ken also demonstrated that using the touch point strategy was more effective for him than using the number line. During the number line strategy phase, Ken’s performance was ascending, but he was unable to achieve 40% accuracy or better. Ken solved single-digit addition problems and reached criterion faster using the touch point method.
Figure 1. Percentage of single-digit addition mathematics problems using the number line and touch points strategies answered correctly by Ashley, Robert, and Ken.
Discussion

The purpose of this study was to replicate and extend a previous study by Cihak and Foust (2008) comparing the effectiveness of the TOUCHMATH program using “touch points” and number lines for teaching single-digit addition problems to students with moderate intellectual disabilities. The results revealed that the students performed better using the “touch point” strategy over the number line in acquiring single-digit addition problem-solving skills. The “touch points” strategy was functionally more effective when comparing the number lines and touch points. In addition, this study also supports previous research studies that have demonstrated that “touch points” have the potential to be an effective intervention to teach single-digit addition problems to students with a variety of disabilities (Kokaska, 1975; Scott, 1993; Simon & Hanrahan, 2004; Wisnieski & Smith, 2002). However, no previous research besides the Cihak & Foust study was found that could find any comparison differentiating instructional strategies using the TOUCHMATH program and number lines, or any other interventions, for teaching computational skills to students in middle school with moderate intellectual disabilities. While there were differences in the students learning and differences in how long it took them to obtain their acquisition of the strategies, all three of the students showed the “touch point” strategy was more effective.

However, there are several limitations in this study that need to be addressed. First, the study employed a single-subject design and only examined single-digit mathematics problems using only three students with moderate intellectual disabilities in a self-contained classroom in a middle school, which limits the generalizability of the intervention to larger populations. So, larger samples must be investigated before broad conclusions can be made and more rigorous treatment-control designs are needed. Second, prior knowledge of the TOUCHMATH program was unknown at the time of this study and with the carry over effects that were noted with Ashley, the potential of this prior knowledge can alter the outcome of the study. If the students were exposed to using the TOUCHMATH program in elementary school, what happens to their ability to use the “touch points”? Do the students with various disabilities need to be constantly reminded? Do they need to be refreshed in the “touch point” methods? For example, with Ken, he was quickly able to achieve high scores after the instructional sessions using the “touch points” strategy. Perhaps more longitudinal studies are needed using the TOUCHMATH program with students with a vast array of disabilities beginning in the elementary schools and following the students throughout their secondary grade levels.

This was the beginning skills in using TOUCHMATH and further acquisition skills would include counting-on rather than the counting-all method that was performed in this study. In addition to learning these basic beginning skills obtained using the TOUCHMATH program, students are eventually using their skills and adding or subtracting without the use of the dots on the numbers. This phase of the TOUCHMATH program was not examined in this study. Using the dot-notation method allows those students who struggle with rote memorization to use the count-all or count-on strategy. This strategy also is more feasible to use out in the public, for example, while shopping and purchasing grocery items as there is no other manipulative to carry around such as a number line or blocks, which often can be cumbersome and uncomfortable to carry around as a manipulative.

Future research is warranted to investigate the learning of other mathematical problem-solving skills, such as multiple-digit addition and subtraction problems with and without regrouping and multiplication and division using the TOUCHMATH program for students with a variety of diverse learning and academic disabilities. In addition, further research is needed to explore the use of the TOUCHMATH program for students with and without disabilities in self-contained and inclusive classroom settings across a range of age, grade, and disability categories.

References


Thordike, R. L., Hagen, E. P., & Sattler, J. M.

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