Athens State University

Athens, Alabama • June 2004

A Foundational Research Base for the TouchMath Program

Presented by

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1. Introduction to TouchMath

This paper will present the psychological and educational research related to the TouchMath program, and provide a foundational research base for implementing the program. TouchMath (Innovative Learning Concepts, Inc.) is a multisensory approach to teaching addition, subtraction, multiplication, and division. This multisensory approach meets the recommendations by Gardner (1993) who has suggested a variety of approaches to match the variety of multiple intelligences in the classroom. Learners see the numerals, touch the Touchpoints, say the numbers, and hear the problems as they say them aloud. Levels of representing knowledge - concrete, pictorial, and symbolic, as proposed by Bruner (1966), are also applied with TouchMath. This approach to teaching computation connects the concrete level (manipulatives) and symbolic level (abstract) concepts. Dots, called Touchpoints, are placed on the numerals one through five. Touchpoints with circles appear on the numerals six through nine. These points are counted twice. The Touchpoints help to decrease the abstract and confusing nature of numerals. The Touchpoints also make the number value evident by providing a visual of the number (how many) on the numeral (symbol). Oral and counting strategies are prerequisites. Slower learners receive extra assistance with the computation process. All learners benefit from the visual clues that the program offers. Visual clues and Touchpoints are removed one step at a time, to aid in the transition to the regular textbook. Problems and answers are repeated aloud, to reinforce auditory practice. The following sections discuss the research literature and its relationship to the TouchMath program.

2. The National Council of Teachers of Mathematics (NTCM) - Selected Principles and Standards Relating to TouchMath

The National Council of Teachers of Mathematics (NCTM) is the leading mathematics organization, promoting standards and principles in mathematics education. The widely accepted and used document, Principles and Standards for School Mathematics (2000), discussed six Principles or themes to guide programs in school mathematics, as well as 10 Standards proposing content and process goals.

The TouchMath program closely meets the NCTM's "Number and Operations Standard." TouchMath's scope and flexibility are intended to:

1. accelerate math comprehension among children as early as ages 4 to 5;
2. build a firm foundation and speed up learning in kindergarten through 3rd grade;
3. remediate learning problems in any regular grade-level classroom;
4. be used in special education with students who have mild to severe learning disabilities;
5. facilitate comprehension among students who have autistic spectrum disorders; and,
6. support remedial math instruction in high school and adult education classes.

The NCTM's web page (http://nctm.org/standards) states that Principles and Standards for School Mathematics "strongly supports the need for 'computational fluency,' for students to have efficient, accurate, and generalized methods for computing. Without the ability to compute effectively, students' ability to solve complex and interesting problems is limited. Other sections take on the relationship between understanding and fluency, arguing that fluency is best developed on a strong conceptual base. This reasoned position, carefully developed with wide input and based on research, provides focus for continued discussion, reflection, and refinement of our classroom practices."

NCTM describes computational fluency as "having and using efficient and accurate methods for computing." Table 1 represents the NCTM's "Number and Operations Standards," which is supported using the TouchMath program:

Table 1	
Number and Operations Standard for Pre-K-2	
Compute fluently and make reasonable estimates.	
Pre-K-2 students should:	• develop and use strategies for whole-number computations, with a focus on addition and subtraction;
	• develop fluency with basic number combinations for addition and subtraction;
	• use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.
Number and Operations Standard for Grades 3-5	
Compute fluently and make reasonable estimates.	
Grades 3-5 students should:	• develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 x 50;
	• develop fluency in adding, subtracting, multiplying, and dividing whole numbers;
	• develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results;
	• develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience;
	• use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals;
	• select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tools.

3. The National Association for the Education of Young Children (NAEYC) - Selected Guidelines Relating to TouchMath

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The purpose of the National Association for the Education of Young Children (NAEYC) is to promote the achievement of healthy development and education for all young children. The following excerpts are from a paper entitled, "Early Childhood Mathematics: Promoting Good Beginnings," and can be found at http://www.naeyc.org. It is a joint position paper of the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM).

"Position: The National Council of Teachers of Mathematics and the National Association for the Education of Young Children affirm that high-quality, challenging, and accessible mathematics education for three-to-six-year-old children is a vital foundation for future mathematics learning. In every early childhood setting, children should experience effective, research-based curriculum and teaching practices. Such high-quality practice in turn requires policies, organizational supports, and adequate resources that enable teachers to do this challenging and important work."

"Rationale: As a society, we are becoming more aware of the importance of early experience in learning to read and write. A similar awareness with respect to mathematics is critical. Early childhood mathematics has a growing knowledge base about learning and teaching as well as an expanding array of research-based curriculum resources. Teachers are eager to provide young children with good beginnings. Now professional preparation programs, education agencies, policymakers, and other partners must mobilize the commitment and resources to apply what we know, support teachers' work, and generate significant progress in early childhood mathematics."

"Recommendations: In high-quality mathematics education for three-to-six-year-old children, teachers and other key professionals should:

1. enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social worlds;
2. build on children's varying experiences, including their family, linguistic, and cultural backgrounds; their individual approaches to learning; and their informal knowledge;
3. base mathematics curriculum and teaching practices on current knowledge of young children's cognitive, linguistic, physical, and social-emotional development;
4. use curriculum and teaching practices that strengthen children's problem-solving and reasoning processes as well as representing, communicating, and connecting mathematical ideas;
5. ensure that the curriculum is coherent and compatible with known relationships and sequences of important mathematical ideas;
6. provide for children's deep and sustained interaction with key mathematical ideas;
7. integrate mathematics with other activities and other activities with mathematics;
8. provide ample time, materials, and teacher support for children to engage in play, a context in which they explore and manipulate mathematical ideas with keen interest;
9. actively introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies;
10. support children's learning by thoughtfully and continually assessing all children's mathematical knowledge, skills, and strategies."

Regarding the NAEYC-NCTM joint statement, Clements wrote, "to achieve high-quality mathematics education, we should enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social world" (2003, 4). The TouchMath program is an effective program for working with addition, subtraction, multiplication, and division. Here are some ways that the TouchMath program relates to the NAEYC's numbered guidelines listed above:

1. Children's dispositions towards mathematics are improved when they are provided with the skills to do efficient computation, as provided with the TouchMath program.
2. This guideline relates to children's abilities to apply the computation to their real lives. When children develop effective skills through the TouchMath program, they are able to focus on the real-life application, instead of the task of computation.
3. The TouchMath program, as shown in this paper, is based on children's development.
4. Children working with the TouchMath program can use their computation skills to engage in problem-solving. With effective methods for computing, children's minds are freer to engage in problem-solving.
5. Important mathematical ideas are supported through the use of the TouchMath program since the computational skills are introduced and reinforced using this sequential program.
6. Children are engaged in deep and sustained computation activities using the TouchMath program.
7. Since children develop effective computation strategies with the TouchMath program, they are able to integrate these strategies into other areas, such as adding inches as a plant grows.
8. Children who engage in the TouchMath program are provided ample time, materials, practice, and teacher support to ensure mastery.
9. The TouchMath program allows students to actively participate in computation activities and apply these strategies in a variety of settings.
10. Teachers using the TouchMath program support children's learning by thoughtfully and continually assessing their knowledge, skills, and strategies.

4. The Link with Bruner's Research and the TouchMath Program
• Concrete, Pictorial, and Symbolic Levels of Representing Knowledge

Appropriate materials and instruction are important for mathematics understanding. Dutton and Dutton (1991) proposed that teaching according to Bruner's theory of cognitive stages should involve moving from the concrete/manipulative level, to the pictorial level, and eventually to the symbolic level. Grouwns (1992) offered that correctly using concrete materials could virtually eliminate mathematics anxiety.

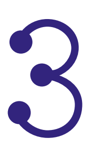
Bruner (1963, 1966) wrote that mental development follows three stages: concrete, pictorial, and symbolic. At the beginning of instruction and in early years, we learn through manipulative activities. After we have mastered this level, we are able to move to the use of pictures, and then, finally, we can rely on symbols to represent the pictures and objects.

The following are the three stages of mental development. The concrete stage is also referred to as the enactive or manipulative stage. Children from the ages of birth through two years are reliant upon this form of learning. In effect, we might say that action is the way infants and young children learn. It is also the best way to learn a concept or procedure no matter what the age level. The concrete stage relates to Piaget's sensorimotor stage, discussed in the next section. Concrete (hands-on) materials are called "manipulatives" and encourage children to use their senses to learn. The pictorial stage is associated with the learning that children of the ages two to seven are capable of exhibiting, since they are able to develop and use different types of mental imagery. At this stage, a child realizes that a picture can represent an object or manipulative. This stage relates most closely with Piaget's preoperational stage. Learners at ages older than seven are also capable of and benefit from comprehending concepts and skills by using mental imagery and pictorial representations. In comprehending, a picture IS worth a thousand words. The most advanced is the symbolic stage, which most closely relates to Piaget's later concrete operational stage and the formal operations stage. Learners ages 7 and older experience this stage. Examples of this stage include using symbols, such as numerals (1,2,3, etc.), alphabetic characters (a,b,c, etc.), and indicators of operations (+, –, x, ÷, =). Each symbol represents a picture, object, or action from the earlier stages.

Figures 1 and 2 show examples of how the TouchMath program matches Bruner's research. TouchMath follows closely with this research since children can use manipulatives to solve the computation problems. They also relate the manipulatives (or counters) to the Touchpoints on each numeral. The very act of touching and counting the Touchpoints provides a concrete experience. In fact, each numeral (3 as an example) is at the highest level of representing knowledge, since it is a symbol. TouchMath bridges the gap between the pictorial and symbolic levels by putting Touchpoints on the numerals to show the number (or quantity). The numeral serves as the symbolic and the Touchpoints serve as the pictorial, providing a picture of the quantity on the symbol.

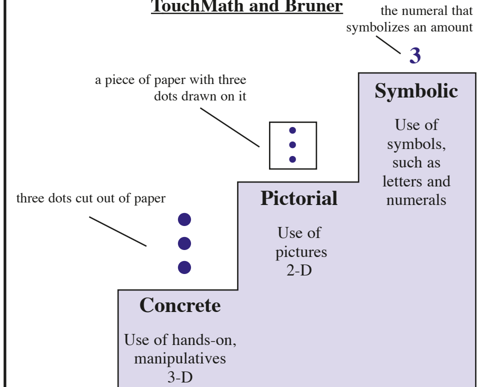
Figure 1

TouchMath combines all three levels, especially the pictorial and symbolic. This provides the student with a support system like no other.



Concrete - Students can touch and point out three dots.
Pictorial - Students can see a picture of three dots.
Symbolic - Students can write and read the shortcut form (symbol) of three as "3".

Figure 2
TouchMath and Bruner



- **The Spiral Curriculum**

Bruner (1960, 1963, 1966, 1996) advocated that students return to a concept in a spiral direction, with sophisticatedly more advanced elements of the concept introduced during successive encounters. His idea was to preplan the curriculum as a learning spiral where “students can revisit the concepts at greater and greater levels of generality and mathematical formality” (Cowan, Morrison, & McBride, 1998, 221). Bruner’s belief was that any content could be taught to children as long as we know the child’s current level of understanding of the content and we then present that content at incremental levels at and just beyond the child’s current level. The TouchMath program is a highly sequential program mimicking the spiral curriculum and details steps for learners along the way. No step is too big since each lesson provides practice and introduces computation skills at increasingly more sophisticated levels. Children using the TouchMath program can access the skills and concepts at entry points, and progress to increasingly more complicated learning in future lessons. For example, plenty of practice is provided with single-digit addition *without* regrouping before moving to single-digit addition *with* regrouping.

Bruner’s research has been well-respected for years throughout the educational literature. Concerning the application of Bruner’s theories today, DeLamater (1999) surmised, “If you look hard, you can find Bruner's ‘spiral curriculum’ reflected in some textbook series based on the National Council of Teachers of Mathematics standards, or catch a glimpse of his idea that teachers need to know their subject matter well transmuted into attempts to define and standardize the competencies of teachers into a form that can be tested by exam. And an echo of Bruner's belief in a vital connection between the university and the schoolhouse can be perceived in the current ‘standards’ movement in the academic disciplines” (40).

5. The Link with Piaget’s Research and the TouchMath Program

Piaget (1975) proposed that people’s development occurs in four stages: sensorimotor, preoperational, concrete operational, and formal operational. During the <u>sensorimotor</u> stage, from birth to about age two, infants learn by their senses and by moving. The only way they know anything is through these modes. The TouchMath program caters to the sensorimotor development by providing Touchpoints on the numerals so that children can experience touching the dots and saying the numbers as they count. Some teachers enlarge numerals and place tactile stimulants, such as fine sand paper or felt dots, on the Touchpoints so that the experience is heightened for children. The <u>preoperational</u> stage occurs at approximately age two and continues through age seven. These children are able to process language as a mode of learning. The TouchMath program provides repetition of effective statements throughout the computation processes. When children learn these statements, they are able to make sense of the visual and hands-on experiences. The <u>concrete operational</u> stage is evidenced in children in grades one through six. These children can engage in logical thinking provided that the computation is accompanied by manipulatives. Using TouchMath, children can relate their classroom manipulatives to the Touchpoints on the numerals. This helps them to bridge the gap between the concrete manipulations and the symbolic representations. The <u>formal operations</u> stage is evidenced in learners from about age 12 and older. These learners can work both concrete (manipulative) situations as well as abstract (symbolic) problems. The TouchMath program facilitates this stage by slowly eliminating the use of the Touchpoints and moving to strictly symbolic notation.

6. The Link with Vygotsky’s Research and the TouchMath Program

- **Scaffolding and the Zone of Proximal Development**

Vygotsky (1962, 1967) theorized two concepts that relate to sociocultural development: scaffolding and the zone of proximal development. In real life, scaffolds enable workers to reach heights otherwise unattainable. At the beginning of construction, many scaffolds are required, with fewer being required toward the completion. Vygotsky's concept of instructional scaffolds is much the same. Children need teachers or more advanced peers to help them learn. The TouchMath program of teaching computation provides many scaffolds since there are visual and verbal clues as well as incremental instruction. The whole program provides scaffolds, such as arrows, bars, and Touchpoints, and then eliminates them as children master the procedures.

The zone of proximal development is another concept proposed by Vygotsky. Figure 3 shows the concept as illustrated on a continuum line (Vinson, 2001). The zone of proximal development (ZPD) is the level at which the child can learn with scaffolds or assistance. Teachers using the TouchMath program can teach children at their appropriate levels, thus promoting understanding of concepts and skills. The visual clues, Touchpoints, and effective computation statements provide students with just the right amount of instructional assistance they need to move them forward in their understanding. It is a waste of time to try to teach something that is far below or far above the child's zone of proximal development. The TouchMath sequential program provides for individualized instruction to meet the needs of students along those various points on the continuum line.

Figure 3
Lev Vygotsky's Concept of ZPD
(zone of proximal development)

task level: too easy	task level: appropriate	task level: too difficult
result: boredom	result: learning	result: frustration
scaffolding: none	scaffolding: some	scaffolding: much

7. The Link with Skemp's Research and the TouchMath Program

- **Procedural Versus Conceptual Understanding**

Skemp (1979, 1989) conceived two types of knowledge that relate to mathematics: conceptual and procedural. Conceptual and procedural knowledge are differentiated throughout the educational literature (Hiebert & Lindquist, 1990). Although this section will show how the two types of learning are different, Ernest (1999) wrote that they also have similarities, by saying, ". . . it takes human understanding, activity, and experience to make or justify mathematics. Much that is accepted as a sign that persons are in possession of mathematical knowledge consists in their being able to carry out symbolic procedures or conceptual operations" (69). The TouchMath program supports both conceptual and procedural learning, as described below.

"Conceptual learning involves the concepts and meanings underlying the operations as opposed to merely applying rules" (Burris, 2005, 8). Davis (1998) claimed that when we understand something, it is partly due to connecting it to something we already know. Connecting knowledge is described as a needed component for understanding, as well as its use and application. The TouchMath program has several elements of connecting. For example, with multiplication and division, concepts of sets or groups are shown through sequence counting (skip counting) in multiplication, and determining how many groups of the divisor are in the dividend in division.

"Procedural learning involves learning processes or algorithms by rote" (Burris, 2005, 8). If someone merely has procedural learning, without the conceptual understanding, then he or she may not know how to apply the rules, cannot construct alternative ways of doing the problem, and lacks a justification for why the problem is worked a certain way. The TouchMath program, with its visual clues and effective procedure statements, allows learners to connect the conceptual understanding to the procedures.

8.The Link with Gardner's Research and the TouchMath Program
• Frames of Intelligence

Gardner (1983, 1991, 1993) proposed that people exhibit individual intelligence strengths. Willis (2001) described it as a movement away from a single IQ score, to a view of intelligence in many ways. Gardner's view of intelligence can be explained as eight frames of intelligence: bodily-kinesthetic, interpersonal, intrapersonal, linguistic, logical-mathematical, musical, naturalistic, and spatial. According to Willis, teachers can more effectively teach when they keep in mind children's intelligence strengths. The TouchMath program has built-in strategies to accommodate the various ways for children to access the content using their intelligence strengths. Other ways to use the program are suggested here to incorporate children's individual intelligence characteristics.

The first is the bodily-kinesthetic intelligence and can be described as the need to learn through physical movement. Thus the TouchMath program incorporates this strength by having children touch and count the Touchpoints to connect the number (how many) with the numeral (the symbol). The interpersonal intelligence is characterized by a desire to understand and interact with others. These children will probably benefit by tutoring a less-capable peer so that the child being tutored meets success with the program. The third intelligence is intrapersonal and relates to those children who are highly developed in their understanding of themselves. These children do well with setting goals for themselves in the TouchMath program and charting their own progress. Children with linguistic intelligence benefit from saying, listening to, reading, and writing the computation statements. The TouchMath program is inherently bonded with the logical-mathematical intelligence since the program is based on sound teaching of addition, subtraction, multiplication, and division. Children with musical intelligence find heightened abilities in anything using the auditory modes, especially of a musical or rhythmic nature. These children benefit from chanting, rapping, or accompanying the computation statements with percussions such as tapping, stomping, and clapping. Naturalistic intelligence is a focus on natural objects, the outdoors, and nature. These children benefit greatly from being given opportunities to glue natural objects, such as leaves and acorns, onto enlarged numerals to represent the Touchpoints. Children with spatial intelligence find the TouchMath program beneficial to their learning strengths since the Touchpoints provide a picture of the number on each numeral. Also, the program is very visually appealing with symbols that provide a picture of what to do. For example, tally marks are used in division to count the number of groups, while dots are used to illustrate the remainder.

9. Preventing or Remediating Mathematics Anxiety Using TouchMath

Mathematics anxiety needs to be avoided with young children. For those who have developed it, ways to reduce that anxiety need to be provided. TouchMath offers easily comprehendible steps for solving addition, subtraction, multiplication, and division problems. Simply put, teaching children these strategies and offering them plenty of practice provides them with confidence in their computation abilities, thus preventing or alleviating mathematics anxiety.

Negative attitudes toward mathematics can produce negative results in mathematics (Dutton & Dutton, 1991; Post, 1992; Wright & Miller, 1981). Many researchers agree that positive practices, such as concrete and pictorial learning, can counteract or prevent mathematics anxiety (Cruikshank & Sheffield, 1992; Furoto & Lang, 1982; Reys, Suydam, & Lindquist, 1995; Van de Walle, 1973). Martinez (1987) wrote that mathematics learning is inhibited by mathematics anxiety, which blocks their learning more than ineffective school curricula or teachers. Smith (1997) characterized students with mathematics anxiety as exhibiting:

a. an uneasiness when asked to perform mathematical computations,
b. an avoidance of math classes until the last possible moment,
c. feelings of physical illness, faintness, dread, or panic,

d. inability to perform on a test, and,
e. utilization of tutoring sessions that result in very little success.

Studies examining preservice and inservice teachers' mathematics anxiety have also been conducted (Chapline, 1980; Kelly & Tomhave, 1985; Kontogianes, 1974; Sovchik, Meconi, & Steiner, 1981). Martinez wrote, "math-anxious teachers can result in math-anxious students" (1987, 117). Sovchik (1996) stated that teachers who have mathematics anxiety often pass this on to their students. In one study, preservice teachers took a mathematics methods class in which heavy emphasis was placed on concrete learning of mathematics concepts and processes using manipulatives (Vinson, 2001). The study found that preservice teachers learned how to teach mathematics with the manipulatives. In turn, they also learned more mathematics while learning to teach with the manipulatives. By the end of the semester, the preservice teachers' mathematics anxiety had been significantly reduced. The results indicated that what they understood, they were able to teach. What they were able to teach effectively would reduce or prevent anxiety in their future students. Using TouchMath, teachers are provided with solid curricula so that children exhibit success with the materials.

10. Common Communication and Representation Systems: Braille and Touchpoints

Children who are blind or have significant problems with sight are generally taught the Braille code, which is a series of dots and dashes in unique locations to represent symbols of language. According to the International Braille Research Center (http://www.braille.org), "Braille is comprised of a rectangular six-dot cell on its end, with up to 63 possible combinations using one or more of the six dots." Braille is a way of accessing numbers for people with sight impairments, just as TouchMath, with its points on each numeral, is a way of accessing numbers for all children without sight impairments. One important way that the two systems are different is that the Braille code uses positions of dots to represent the unique symbolic characters. On the other hand, the TouchMath way of representing numerals uses a corresponding number of dots to represent the amount, for example, the numeral 4 has four dots on it.

Children with and without sight problems have sometimes limited achievement in mathematics. Both Braille and TouchMath can help children acquire mathematical skills. Kapperman and Sticken (2002) found that "many students who are blind are mathematically illiterate and unable to read or write the Braille code of mathematics." They further noted, "without the ability to read and write the symbols that represent mathematical concepts, the field of mathematics is closed to people who are blind" (855). Similarly, without the ability to manipulate numerals, children with sight are also locked out of certain fields.

Kapperman and Sticken (2003) argued that people who have sight impairments should also be included in national efforts to improve mathematics teaching and learning. They contend that people with sight impairments have the same mathematical abilities as their sighted counterparts. However, they lack the means to learn and to demonstrate these skills. The authors' research has shown that these students' teachers often lack instructional competence in the Nemeth Code (the Braille code for mathematics), which in turn leads to students having limited ways to represent mathematics. DeMario (2000) wrote that it is necessary for teachers to know how to transcribe mathematics content into the Nemeth Code in order for students with visual impairments to achieve at high mathematics levels.

Kapperman, Heinze, and Strickens (1997) conducted a review of literature and found that visually impaired students have low levels of mathematical literacy. They believe that this is a result of teachers having low levels of mathematical Braille skills, and, as a result, they may place more emphasis on content that does not involve mathematics. Many studies focus more on the competency of the *literary* Braille code, rather than the *mathematical* Braille code (Johnson, 1996; Mullen, 1990; Rex, 1989; Spungin, 1989, 1996; Stratton, 1996; Wittenstein, 1994; Wittenstein & Pardee, 1996). It is not so difficult to see the comparison between this lack of preparation and the poor representation of visually challenged people in the fields of education and employment requiring advanced mathematical skills.

Kapperman and Sticken (1998) suggested that the Braillewriter is important to use as a calculation tool with blind children. They noted that the abacus and the talking calculator are important tools to use; however, they do not provide the student with the same opportunities that sighted children have. Using written symbols allows the visually impaired student to follow the same arithmetic operations as the sighted student. Doing mathematics with numerals is important for children with and without sight since numerals are the symbolic notations for mathematics. This ability to do mathematics with symbols can be achieved through the help of TouchMath by using the numerals with Touchpoints, or Braille by using such aids as the Braillewriter.

11. The Needs of Visual, Auditory, and Tactile/Kinesthetic Learners Met through TouchMath

According to Dunn and Dunn (1978), there are three basic modes of processing information: visual, auditory, and tactile or kinesthetic. Sarasin (1998) noted that many children prefer to process information visually and can easily be frustrated by a teacher who uses the auditory mode of "telling" in order to teach. These children are visual learners. According to Dunn and Dunn (1978), visual learners process their information primarily through sight. To cater to this type of learner, the TouchMath program provides visual clues, such as arrows and Touchpoints. Some students prefer to listen in order to learn. These children are auditory learners. "These learners are usually verbal in nature, and often tend to think aloud" (Fielding, 1995, 29). Dunn and Dunn (1978) noted that auditory learners process their information primarily through sound, hearing, speaking, and listening. The TouchMath program provides for the learning style of these children by verbalizing the steps to the computation. The kinesthetic learner prefers physically doing something to learn the content. "Tactile or kinesthetic learners learn by doing. Traditionally, this type of learner has been the most neglected in education settings" (Mixon, 2004, 48). Dunn and Dunn (1978) wrote that kinesthetic learners process their information primarily through physically experiencing the information. Barbe and Milone (1980, 1981) maintained that 15% of elementary children are kinesthetically oriented, yet schools are predominately visually and auditorially oriented. In the TouchMath program, children count by touching the Touchpoints and saying the number.

Mixon (2004) wrote to teachers that "by addressing all three learning styles you will help students develop their weaker learning modalities as well as their stronger, more natural ones. Students can then become more versatile learners in varied settings" (48). Friedman and Alley (1984) discussed an instrument to identify auditory and visual linguistic, auditory and visual numerical, audio-visual-kinesthetic combination, individual or group learner, and oral or written expressive learning styles. Corno and Snow (1986) wrote that "the success of education depends on adapting teaching to individual differences among learners" (605). The TouchMath program provides for each of these types of learners.

12. Facilitating Concepts of Number, Numeral, and Number Words with TouchMath

"A *number* indicates a quantity. A *numeral* is the symbolic written expression of a number. For example, the word *five* represents a number that indicates a quantity; the numeral for this quantity is 5" (Burris 2005, 63). Sherman, Richardson, and Yard stated, "numeric symbols are connected to materials by writing the numerals that represent the quantity in the presence of manipulatives. Number words are connected to numerals and materials that represent a quantity" (2005, 36). The TouchMath program creates an even better connection between the number and the numeral, since the numeral has the Touchpoints that represent the number on it. Effective computation statements are provided that include a number word for each numeral as it is read to carry out the computation. Posters and activity pages within the kits provide a clear connection between the numeral and its corresponding number and number word.

Several writers have documented the importance of using mental imagery with quantities in order for children to develop number concepts (Baroody & Standifer, 1993; Payne & Huinker, 1993; Van de Walle 1990; Wirtz 1980). Each discussed how the dot patterns, such as the ones on dice (number cubes), dominoes, and frames (for five and ten), were necessary for children to form a concept of number. Kline (1998) suggested using dot patterns and ten frames to help children develop mental imagery and a sense of number. She noted that Kliman and Russell (1998) had developed a curriculum using quick-image recognition of dot patterns with children in first and second grades. Touchpoints on the numerals serves to aid in such mental imagery.

13. Enhancing Counting, Cardinal Number, and Numeration Skills through the Use of Touchpoints

Many children come to school with the ability to orally count to 10 and higher (Fuson, Richards, & Briars, 1982). Using manipulative activities, children construct the meaning of counting and numeration (Van de Walle, 1990). Children using the TouchMath program primarily use it for the four computation processes: addition, subtraction, multiplication, and division. However, the concepts of counting, cardinal number, and numeration are strengthened and facilitated while using the program. When children touch and count the Touchpoints on the numerals, they are practicing their counting skills. One-to-one correspondence is provided through the touching of the dots on the numerals. The concept of cardinal number is strengthened as children determine the total number of Touchpoints on each numeral. Cardinal number is the total number in the set, and is the last counting word said. When children touch and count the Touchpoints on the numeral 4, they see four Touchpoints, and say the words to accompany each number, which strengthens their sense of numeration. It is important to note that children need plenty of practice with the steps of touching and counting the Touchpoints. As noted by Troutman and Lichtenberg (1995), children sometimes exhibit counting errors when they are trying to determine the sum of two numbers.

14. TouchMath's Strategies for Recalling Number Facts

Burton and Knifong (1982) suggested that teachers who help children master the basic facts empower them to grasp the meaning of computation as well as the real-life situations in which they will use those facts. Heege (1985) wrote that when children learn basic multiplication facts, they use acquired knowledge. If teachers encourage children to use informal thinking strategies, then the gap between figuring out the answer and knowing it by heart eventually closes. This helps them to think of computation as more than a set of rules. TouchMath is one such program that empowers children to master the basic facts and computation. After basic facts and computation are mastered, children using the TouchMath program can apply these skills to an endless variety of real-world problems.

According to Polya (1957), mathematics is made up of both information and “know how.” However, Cowan, Morrison, and McBride (1998) point out, “schools focus on the transmission of information to their charges and often neglect the most important area, ‘know how.’ For students to retain information, they must understand and internalize the underlying principles” (206). Bruner (1960) emphasized that “computational practice may be a necessary step towards understanding conceptual ideas in mathematics” (29).

One report of an intervention program to help first through fifth grade students learn, recall, and retain the basic facts, resulted in possible causes for students having difficulties (Haught, Kunce, Pratt, Werneske, & Zernel, 2002). Noted causes were lack of time devoted to practicing the basic facts at school, practice at home that was inconsistent, and minimal emphasis on basic facts within the textbooks. The results indicated that regardless of games or music used to practice the basic facts, students' scores on timed math tests improved when children were provided with increased and sustained practice. Fuson and Brinko (1985) conducted a study with children in grades two through four during a period of six weeks. Children used either flash cards or a computer program in order to practice basic mathematics facts. The results showed that equivalent learning occurred. These are examples of research that show the importance of sound computation procedures and plenty of practice. The TouchMath program provides such sufficient practice and strategies for determining the basic facts.

After much study, Baroody (1999) suggested that practice is an important part of children automatizing their thinking. Other researchers have suggested that retrieval strategies are not supplanted by thinking strategies (Jerman, 1970; Siegler, 1987). Several researchers have found that adults also use many strategies to answer problems involving combinations of basic facts (LeFevre, Bisanz, Daley, Buffone, Greenham, & Sadesky, 1996; LeFevre, Sadesky, & Bisanz, 1996). In an earlier study, Baroody (1985) argued that children do not store hundreds of basic number facts as separate pieces of information. Instead, they learn them by processing relationships. The TouchMath program encourages the use of relationships among numbers since multiple practice is provided with sets of problems that follow the same rules. For example, addition problems resulting in an answer of 10 are provided in sets so that children can see the multiple ways to make 10 (6 + 4, 7 + 3, etc.).

15. Mathematical Communication Facilitated through TouchMath

One of NCTM's process standards is the "Communication Standard" (NCTM, 2000). Communication is a means of understanding the computation and sharing that understanding with others. "An important factor in the warranting of knowledge is the means of communicating it convincingly in written form, i.e., the rhetoric of mathematics. Skemp's concept of 'logical understanding' anticipates the significance of tacit rhetorical knowledge in school mathematics" (Ernest, 1999, 67). Kitcher (1984, 1991) contended that mathematics involves a "set of accepted statements." These are comprised of language and the interpretation of mathematical symbols. The TouchMath program has effective computation statements that are practiced and can turn the manipulation of the symbols into comprehendable processes. Rotman (1993) also pointed out that mathematics is expressed with language, and in effect, mathematics could not exist without language. An important aspect of the TouchMath program is that it provides both teachers and students with ways to express the computation so that each step makes sense.

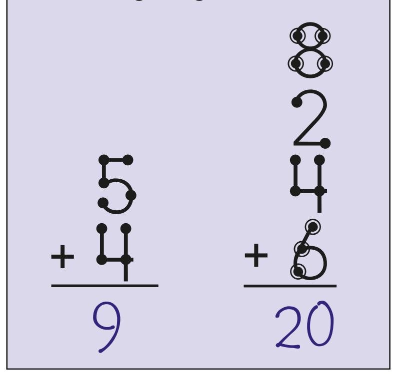
Students use communication to help them develop mathematical understanding (Pugalee, 1999). "The communication processes of reading, writing, speaking, and listening are important for achievement in any content area" (Muth, 1997, 772). Several researchers have demonstrated the importance of verbalizing and discussion among students (Noddings, 1985; Peterson & Swing, 1985; Webb, 1984, 1985, 1991). The NCTM (1989) encourages communication in order to clarify thinking, as well as refine it. According to the National Research Council (1989), people need *mathematical* literacy (or numeracy, as the British call it), as well as *verbal* literacy. The Department for Education and Employment (1998) stated that people need to develop mathematical literacy (or numeracy) in order to have full use of the curriculum and, eventually, the adult world of work. They describe numeracy as the knowledge of numbers and number operations. The Mathematical Sciences Education Board (1996) stated that representing mathematical information involves multiple concepts including numerical, algebraic, graphical, as well as verbal modes.

According to Vygotsky (1962) conceptual understanding is the relationship between thought and language. All language is based on social interaction, since it takes at least two people to communicate. Therefore, communication facilitates mathematical understanding. Discourse and communication are important to conceptualizing mathematics, even if the communication is written instead of verbal (Pugalee, 1995, 1997). Verbalizing steps to computation problems is vital since the language of mathematics needs to be internalized. TouchMath provides repeated steps and common language throughout each of the related problems. For example, the same strings of steps are used to describe all single-digit multiplication problems. Children are encouraged to say aloud the problem, the steps, and the answer. Since the same language is repeated, children are able to learn the appropriate language associated with the steps. When children are encouraged to repeat the problems and the answers, they receive reinforcement in communication – which is an aid to conceptual understanding.

16. Addition with TouchMath

• Beginning Addition

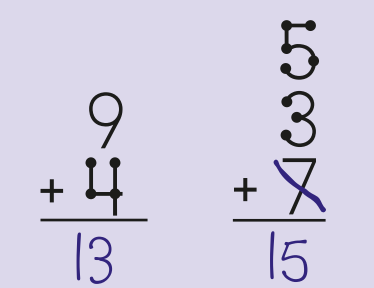
Figure 4
Beginning Addition



Prerequisites to TouchMath addition methods are abilities to count, recognize numerals, and write two-digit numbers. Children are encouraged to touch each point with their pencils and count. In the first problem, shown in Figure 4, students start with the 5 and count each Touchpoint. Next, they count "six, seven, eight, nine" while touching the points on the 4. They write the answer and repeat the problem with its answer aloud. On the next problem, notice that there are four addends. This is a plus for the TouchMath program, because children can continue counting to find the sum, even with more than two addends. Children touch the eight points on the 8, move to the 2 and count "nine, ten," move to the 4 and count "eleven, twelve, thirteen, fourteen," and move to the 6 and continue counting up to the total of 20.

• Addition with Continuance Counting

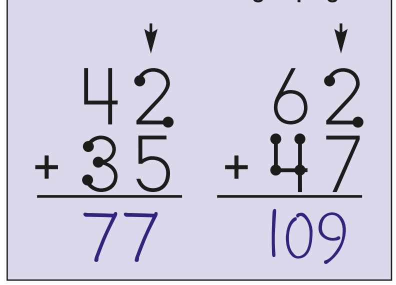
Figure 5
Addition With Continuance Counting



Continuance counting means to start with the largest number and count up from that number. In the last problem we discussed (8 + 2 + 4 + 6), where children counted each of the eight Touchpoints. With continuance counting, counting all eight would not be necessary. The children will touch the largest number, say its name, and continue counting. Notice in the first problem, in Figure 5, the points are removed from the 9. In this problem, the children say "nine" (touching the 9) and count "ten, eleven, twelve, thirteen" (counting the points on the 4). For the next problem, notice that there are three addends and the 7 is crossed out. Children will say, "I cross out the largest number, say its name, go to the top of the column, and continue counting."

• Addition without Regrouping

Figure 6
Addition Without Regrouping

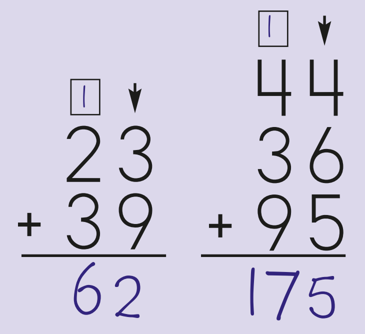


The statement that children repeat while doing this two-digit problem is, "I start on the side with the arrow. The arrow is on the right side." This is necessary because words and multi-digit numbers are read from the *left*. However, multi-digit addition problems are solved from the *right*. The arrow serves as a visual clue. In the first problem, in Figure 6, children start on the right side, and say "five" (pointing to the largest number – 5) and "six, seven" counting the Touchpoints on the 2. Then they move to the tens place and add those. They should be encouraged to read the problem and answer to reinforce reading and recognizing two-digit numbers.

• Addition with Regrouping

Another visual clue that is added to the process of addition with regrouping is the box, as shown in Figure 7. Children are encouraged to say the arrow statement, "I start on the side with the arrow. The arrow is on the right side." The answer to the first column on the first problem is 12. They put the 1 (or one ten) in the box and the 2 below. Writing the tens first, helps to eliminate reversals.

Figure 7
Addition With Regrouping



• Addition Errors Prevented and Remediated with TouchMath

When children add two-digit numbers with zeros, they sometimes fail to complete the addition using the zeros (Troutman & Lichtenberg, 1995). Sherman and colleagues (2005) reported three main addition problems that elementary children experience: failing to regroup (recording a two-digit answer in the ones place), completing addition from left to right (16 + 17 = 113), and adding all the numbers together (14 + 3 = 8) in a two-digit problem as if they were all ones (Sherman, Richardson, & Yard, 45, 50, 53). When children add a number with multiple addends, they sometimes make mistakes since there are more than two numbers to deal with (Troutman & Lichtenberg, 1995). Since addition is a binary process, meaning that only two numbers can be added together at once, then the TouchMath program facilitates children's abilities to add multiple addends with ease. Adding the numbers 9 + 2 + 6, the 9 and 6 will be added to make 15. The 9 will receive a strike through it to show that it is the number representing the most. Counting up from 9 by 6 will result in 15. After that, the child would count up 2 from 15. Striking through the 9 allows the child to keep up with which addends were added. Children can more readily see the transitivity of equality (Troutman & Lichtenberg, 1995) by focusing on the Touchpoints on each number. Transitivity means that 4 + 2 = 6, just as 3 + 3 and 1 + 5 equal 6. Either way it is shown, the numerals with the corresponding Touchpoints will allow the child to see a total of 6. In a similar way, children can "see" that 4 is greater than 2 because 4 has more Touchpoints than 2 does. Two of TouchMath's visual clues allow children to more readily answer the problems without errors. The box is a visual clue that provides a "container" for writing the regrouped digits. The other visual clue is the arrow, which accompanied by its statement, allows children to know where to start the computation process.

17. Subtraction with TouchMath

• Beginning Subtraction

A prerequisite to TouchMath subtraction is backward counting. Students should be able to count backward on the number line from 18. For example, using the problem, 48 – 29 = 19, when we regroup for the ones place, the problem is 18 – 9. Therefore, 18 is the greatest number to count back from since 18 – 9 = 9. In the first problem shown in Figure 8, Touchpoints are on the bottom number only. The subtraction statement is, "I touch the top number, say its name, and count backward using the Touchpoints. The last number I say is the answer." Remember that, as in addition, students should repeat the problem and the answer to reinforce basic facts. Using the second problem, the student might say, "I touch the top number. The top number is 14. I count backward using the Touchpoints on the 6. Thirteen, twelve, eleven, ten, nine, eight. Eight is the answer. Fourteen minus six equals eight."

Figure 8
Beginning Subtraction

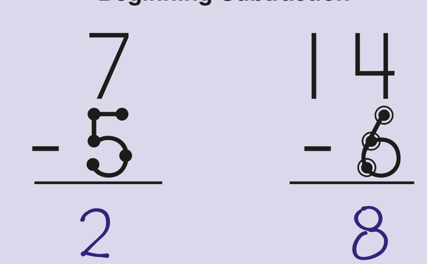
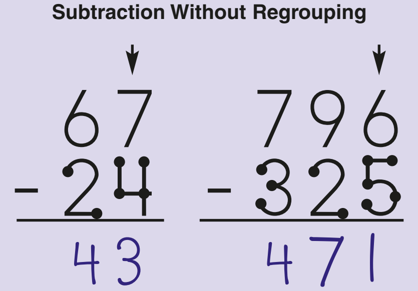


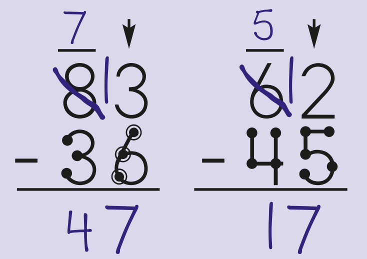
Figure 9
Subtraction Without Regrouping



• Subtraction without Regrouping

When we get to two-digit subtraction without regrouping, the process is the same for each place value – tens and ones, as shown in Figure 9. An arrow serves as a visual clue, as it did in addition, to show students which side is their starting place. Using the arrow statement, "I start on the side with the arrow. The arrow is on the right side," students receive reinforcement for starting on the appropriate side. The reason the arrow is necessary at the beginning is that when we read words and multi-digit numbers, we start on the left. When working computation problems that are vertical, we start on the *right*. The students might do the first subtraction problem using the arrow statement and the subtraction statement by saying, "I start on the side with the arrow. The arrow is on the right side. I touch the first number, say its name, and count backward using the Touchpoints. Seven is the top number. Six, five, four, three. I write the three and continue to the next column."

Figure 10
Subtraction With Regrouping



• Subtraction with Regrouping

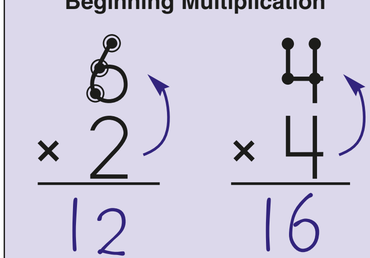
In subtraction with regrouping, notice that a new visual clue has been added, as shown in Figure 10. This "bar" allows students to have a place to put the regrouped set(s) of ten. The regrouping statement is, "I must borrow or regroup if I cannot continue to count backward using all the Touchpoints." So a student doing the first problem might say, "I start on the side with the arrow. The arrow is on the right side. I start with the top number, 3, and count backward using the Touchpoints on the 6. Since I cannot continue to count backward, I must regroup. I will cross out the 8, place the 7 on the bar above the 8, and place my 1, or 1 ten, beside the 3." The student will then follow the aforementioned steps to complete the problem. Notice that the regrouped 1 in front of the 3 is drawn as large as the 3.

• Subtraction Errors Prevented and Remediated with TouchMath

"Learning how to reason sensibly about two-digit numbers requires children to think about relationships between tens and ones, particularly when subtracting with regrouping. If children develop procedures for adding and subtracting without understanding why those procedures work, they learn that mathematics is about performing procedures without reasoning, not about reasoning with numbers" (Whitenack, Knipping, & Underwood, 2001, 233). Nagel and Swingen (1998) encouraged children in a study to explain their thinking out loud when doing addition and subtraction problems. Those children who demonstrated comprehension of the computation were those who explained the problem's solution in terms of place value, especially when the problem required regrouping. Bartek (1997) noted that second and third grade children often have trouble when the addition and subtraction problems require regrouping, and when they have to work across zeros. She further commented that the traditional textbook methods are often difficult to teach. Sherman and colleagues (2005) reported three main subtraction problems that elementary children experience: always subtracting the smaller number from the larger (86 – 7 = 81), regrouping whether the problem requires it or not, and failing to regroup the tens place value in a three-digit problem when the tens place is a zero (Sherman, Richardson, & Yard, 69, 74, 77). The TouchMath program provides two visual clues for students. The bar allows children to have a "shelf" for putting their regrouped digits. The arrow shows students where to start the process. Effective computation statements, which are practiced with each problem, allow children to comprehend the processes.

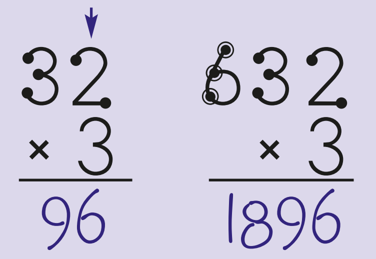
18. Multiplication with TouchMath
• Beginning Multiplication

Figure 11
Beginning Multiplication



In the processes of addition and subtraction, forward and backward counting were prerequisites. In multiplication, skip counting is a prerequisite. Skip counting, or sequence counting, builds on the skill of counting. Later it will help with counting nickels, dimes, sets, and working with base systems, and square roots. The TouchMath program recognizes that skip counting can be taught in numerical order, starting with the 2s, 3s, 4s, and so on. However, they recommend skip counting by 2s and 5s first, and then by 3s, 4s, 6s, 7s, 8s, and 9s. Look at Figure 11. The multiplication statement is, “I sequence count by the bottom number, while touching the Touchpoints on the top number. The last number I say is the answer.” The child doing the first problem might say, “I sequence count by 2, while touching the Touchpoints on the 6. Two, four, six, eight, ten, twelve. Twelve is my answer. Two times six equals twelve.” As in addition and subtraction, it is important to repeat the problem and the answer. This way they begin to remember basic facts. Notice that we start with the bottom number when multiplying, because this is how it will be done when we have multi-digit multiplication problems.

Figure 12
Multiplication Without Regrouping

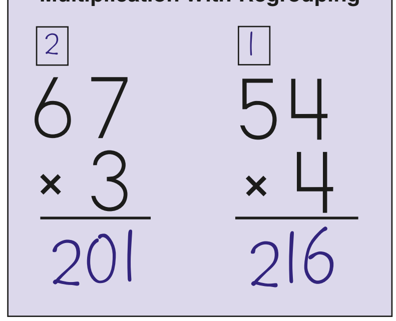


• Multiplication without Regrouping

Look at Figure 12. Students have thus far used the arrow on the right side to help them remember that their starting point is on the right. This is the opposite of how we read words and multi-digit numbers. If necessary, put the arrow as a visual clue and review the arrow statement, “I start on the side with the arrow. The arrow is on the right side.” This problem is worked by using the same processes described in beginning multiplication. The child working this problem might say, “Three, six (touching the Touchpoints on the 2). Three, six, nine (touching the Touchpoints on the top 3). Thirty-two times three equals 96.”

• Multiplication with Regrouping

Figure 13
Multiplication With Regrouping



In the addition process, the box was a visual clue, and served as a container for the regrouped number. In the multiplication with regrouping process, shown in Figure 13, the box serves as a container for the ten (or tens) that represent a two-digit product from the ones place. We always add what is in the box. We will talk this first problem through. “I count by 3 on the 7. Three, six, nine, twelve, . . . twenty-one. I write the 2 in the box and the 1 below the 3.” The process continues with the multiplication of 3 x 6 and adding the 2 in the box. Notice that when writing 21 as the product from the ones place, the 2 is written in the box before the 1 is written below the 3. This is done because it is the way we write two-digit numbers – ten first, then ones. This also helps with the problem of “I forgot to carry.” Also, the Touchpoints have disappeared from the numerals of these problems. The reason is that by this time students probably touched the Touchpoints on the numerals enough to know where they are without having them visually represented. Since the goal of TouchMath is to eventually have students transition to textbooks and other curriculum materials, the visual clues are used until they become automatic, and then disappear from future problems. Another thing to notice is that the problems and answers have been repeated enough, that, at this point, students may know the number facts and not have to sequence (skip) count to get the products.

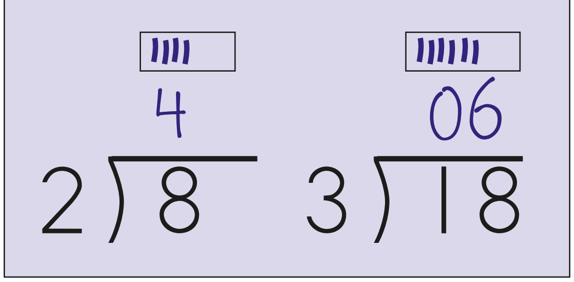
• Multiplication Errors Prevented and Remediated with TouchMath

Troutman and Lichtenberg (1995) wrote “some children can learn to do ‘skip counting’ as early as the first grade. Then, with appropriate and thoughtful encouragement, counting by twos, fives, and tens seems to come easily. Counting by threes can be accomplished with a little more effort” (236). This provides support for TouchMath’s use of sequence (skip) counting for multiplication problems. Sherman and colleagues (2005) reported three main multiplication problems that elementary children experience: using the partial product recorded above the problem for more times than is intended (during multiplying a two-digit factor by the ones place and then using it again during the multiplication by the tens place), failure to record partial products (17 x 5 = 55), and adding before multiplying (Sherman, Richardson, & Yard, 92, 98, 101). For example, the last problem could be illustrated with 17 x 5 where the child records the answer as 205. The child's thinking is "7 x 5 equals 35, record the 3 above the tens place and the 5 where the ones answer goes." The tens multiplication proceeds without adding the regrouped 3. The TouchMath program for multiplication provides effective computation statements that are repeated throughout the practice. The arrow serves as a visual clue and also helps children know where to start the problem. During the multiplication with regrouping process, children are instructed to write the tens place first and record the ones place in the answer. This keeps them from forgetting to regroup. For children who have trouble aligning their problems, the TouchMath program provides statements that are practiced so that children know where to record what.

19. Division with TouchMath

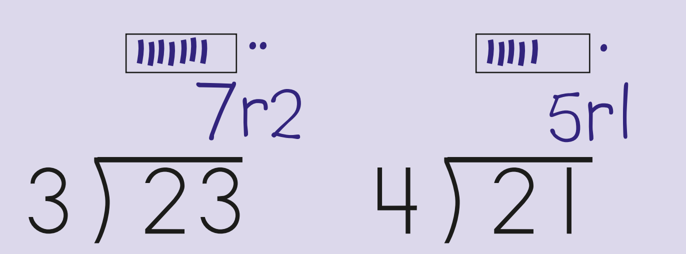
• Beginning Division

Figure 14
Beginning Division



The first thing to notice about the division problems, as shown in Figure 14, is that there are no Touchpoints. By this time, students have completed the processes of addition, subtraction, and multiplication using the Touchpoints. Although they may continue to touch and count on the numerals, the visual dots are usually not necessary. They have memorized where the Touchpoints are and what they represent. The division statement is, "I sequence (skip) count by the divisor and get as close to the dividend as I can without going over the dividend." Tally marks are placed in the box while the students skip count. A student completing the first problem might say, "I sequence count by 2 (the divisor) and get as close to the 8 (the dividend) as I can without going over the 8. Two, four, six, eight (putting four tally marks one at a time in the box). I will count the tally marks from the box. There are four. I will write 4 above the 8. Eight divided by two equals four." Remember to continue to repeat the problem and the answer to reinforce auditory learning and to practice basic facts: "8 ÷ 2 = 4." The second example problem shows a two-digit dividend. Since we cannot sequence count by 3's to get 1, we put 0 above the 1 in 18. The division process then continues with the sequence counting and tally marks as noted above.

Figure 15
Division With Remainders



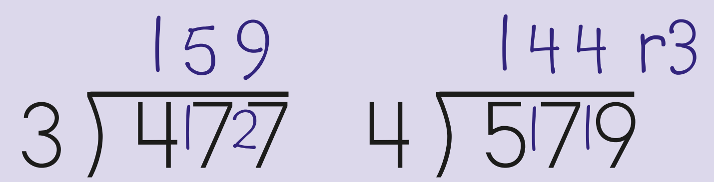
• Division with Remainders

In the first problem in Figure 15, we progressed to the point that students understand what it means to put a 0 above the 2. Therefore, we discontinue use of the zero as a placeholder to the left of the number. The answer to the first problem is 7 remainder 2, seven tallies inside the box, and two dots outside the box. The tallies represent groups of 3 (the divisor) and the two dots represent the remainder. It would be counted like this, "three, six, nine, twelve, fifteen, eighteen, twenty-one (tallies), twenty-two, twenty-three (dots)."

• Short Division

In short division, students use fact recall. Short division should be thought of as a shortcut method, and is encouraged once students are fluent with their facts, since oftentimes students get lost in the steps of long division. In the first problem in Figure 16, students write a 1 above the 4 because you can sequence count by threes once and have 1 left over. So, they write the 1 beside the 7. Now how many times can you sequence count by threes to 17? Five times, so they will write the 5 above the first 7 and the 2 beside the next 7. The 2 is the difference between 15 and 17. Next, how many times can you sequence count by threes to 27? The answer is 9, so they will write a 9 above the last 7. Finally, they should repeat the problem and the answer. This takes a lot less paper, doesn't it?

Figure 16
Short Division



• Division Errors Prevented and Remediated with TouchMath

Cobb and Merkel (1989) and Thornton (1990) wrote that students find basic fact practice and simple computation easier when they develop effective thinking strategies. Gluck (1991) reported that children often have problems understanding place value when they work with numbers; however, using both concrete and symbolic representations can help them construct the needed concepts. Troutman and Lichtenberg (1995) wrote that emphasis should be placed on the meaning of division and then to proceed slowly through the steps. Sherman and colleagues (2005) reported three main division problems that elementary children experience: reversing digits in the quotient (recording the quotient from right to left), failing to record zeros in the quotient, and failing to record remainders (Sherman, Richardson, & Yard, 117, 122, 128). The TouchMath program has effective computation statements that allow students to comprehend the processes. Using tally marks for the number of groups of the divisor and dots as the number of ones in the remainder allows the child to more easily grasp the concept of the quotient (or partial quotient) in the division problem. The TouchMath's way of limiting what is written in the division problem keeps the child from experiencing problems with keeping the problem aligned. In fact, children can focus more on the quotient than the intermediary steps and symbols to attain it.

20. TouchMath's Transition Capabilities: Transition to More Advanced Skills and Transition to Traditional Curriculum Materials

The TouchMath program is a specialized means of helping children construct addition, subtraction, multiplication, and division skills. The first implementations of TouchMath, in the program's early days, were with special needs learners. The goal was to help these children transition to more advanced skills, so that they could meet the curriculum objectives of their age-level peers. The program allowed special needs learners to receive remediation and specialized instruction so that they could function with their peers' levels of computation in the regular classroom. A second goal was to assist these special learners in moving to the use of the curriculum materials used with their age-level peers or to the materials that these special learners would use in future studies. It did not take long for teachers to recognize that the TouchMath program was effective for all learners, not just those in need of specialized and remedial instruction. TouchMath's current transition capabilities are for all children to construct the skills and concepts so that they can transition to more advanced skills, as well as to eventually transition to symbolic notation in traditional curriculum materials, often used in later grades.

In a recent attempt by this author to determine transition capabilities to more advanced skills, the present author individually conferenced with five K-2 students who had been using the TouchMath program, none of whom had added numbers using more than two addends. Each was asked to show how to add 6 + 7 using TouchPoints. The purpose for the investigation, from the students' viewpoint, was to informally demonstrate the method to the author. In each case, when the students were presented with the scenario of adding 6 + 7 + 8, and asked it they could do this problem, these students continued to count across using the TouchPoints and were surprised that it worked with three addends as well as two. These children were matched with five K-2 students who had not used the TouchMath program. It was found that only the second grader could work the problem with three addends. She revealed that her mother had shown her how to do the problem by writing the subtotal of the first two numbers and then adding the subtotal to the third number. The other children said that they had not learned how to do problems with so many numbers yet. In the case of the children who used the TouchMath program, the children were able to transition to problems with multiple addends with ease.

21. Summary of Research

The purpose of this paper was to relate the research literature to the TouchMath program. Although every important research document and every important researcher was not named in the present research, the most important ones were reviewed and determined to fit well with the TouchMath program. Two important education organizations, the National Council of Teachers and Mathematics and the National Association for the Education of Young Children, were found to have standards and guidelines that TouchMath meets. Different elements of the research conducted by Bruner, Piaget, Vygotsky, Skemp, and Gardner were also related to the TouchMath program. The ways that different learners have their computation needs met were discussed with the categories of visual, auditory, and tactile/kinesthetic learners. Ways that the TouchMath program helps children with numbers, numerals, number words, counting, cardinal number, numeration skills, recalling basic facts, communication, addition, subtraction, multiplication, and division were presented in this paper. The relationship between two systems, Braille and Touchpoints, were linked as effective symbolic representation systems. The ways that the TouchMath program fosters children’s transition to more advanced skills and to traditional curriculum materials were also discussed.

22. Future Research

This paper demonstrates the many ways that the TouchMath program is based on sound, well-respected, and broad-based research. Future research efforts could determine the achievement effects of sustained use of the TouchMath program for computation, as compared to students using other methods or programs. Due to the widespread use of TouchMath, and veteran teachers’ individual choices to use TouchMath instead of other programs, it is expected that both attitudes toward using the program and achievement levels will be more positive than for the group not using the program.

23. References

Barbe, W.B., & Milone, M.N. (1980). Modality. *Instructor, 39*(6), 45-47.

Barbe, W.B., & Milone, M.N. (1981). What We Know about Modality Strengths. *Educational Leadership, 38*(5), 378-380.

Baroody, A.J. (1985). Mastery of Basic Number Combinations: Internalization of Relationships or Facts? *Journal for Research in Mathematics Education, 16*(2), 83-98.

Baroody, A.J. (1999). Children’s Relational Knowledge of Addition and Subtraction. *Cognition and Instruction, 17*(2), 137-175.

Baroody, A., & Standifer, D. (1993). Addition and Subtraction in the Primary Grades. In Jensen, R. (Ed.), *Research Ideas for the Classroom: Early Childhood Mathematics.* New York: Macmillan.

Bartek, M.M. (1997). Hands-On Addition and Subtraction with the Three Pigs. *Teaching Children Mathematics, 4*(2), 68-71.

Bruner, J. (1960). *The Process of Education: A Landmark of Educational Theory*. Cambridge, MA: Harvard University Press.

Bruner, J.S. (1963). *The Process of Education*. New York: Vintage Books.

Bruner, J.S. (1966). *Toward a Theory of Instruction*. Cambridge, Mass.: Harvard University Press, Belknap Press.

Bruner, J. (1996). The Culture of Education. Cambridge, MA: Harvard University Press.

Burris, A.C. (2005). Understanding the Math You Teach. Upper Saddle River, NJ: Pearson Education.

Burton, G.M., & Knifong, J.D. (1982). Learning the Facts: It Need Not Be Frustrating. Elementary School Journal, 83(2), 149-154.

Chapline, E. (1980). Teacher Education and Mathematics: Program Development and Evaluation. Washington, DC: Women's Educational Equity Act Program. (ERIC Document Reproduction Service No. ED 186 272, 13 pages.)

Clements, D.H. (2003). Math: A Civil Right. Early Childhood Today, 17(4), 4.

Cobb, P., & Merkel, G. (1989). Thinking Strategies: Teaching Arithmetic through Problem Solving. In Trafton, P.R. (Ed.), New Directions for Elementary School Mathematics: 1989 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics. 70-81.

Corno, L., & Snow, R.E. (1986). Adapting Teaching to Individual Differences Among Learners. In Handbook of Research on Teaching. Ed. M.C. Wittrock. New York: Macmillan.

Cowan, P., Morrison, H., & McBride, F. (1998). Evidence of a Spiral Curriculum Using a Mathematical Problem-Solving Tool. Interactive Learning Environments, 6(3), 205-224.

Cruikshank, D.E., & Sheffield, L.J. (1992). Teaching and Learning Elementary and Middle School Mathematics. Columbus: Merrill, 24.

Davis, D. (1998). Understanding and Holism. Journal of Philosophy of Education, 32(1), 41-55.

DeLamater, W.E. (1999). An American Classic at 40. Education Week, 19(16), 39-40.

DeMario, N.C. (2000). Teachers' Perceptions of Need for and Competency in Transcribing Braille Materials in the Nemeth Code. Journal of Visual Impairment & Blindness, 94(1), 7-14.

Department for Education and Employment. (1998). Numeracy Matters: The Preliminary Report of the Numeracy Task Force. London: Crown Copyright.

Dunn, R, & Dunn, K. (1978). Teaching Students through Their Individual Learning Styles. Englewood Cliffs, N.J.: Prentice-Hall.

Dutton, W.H., & Dutton, A. (1991). Mathematics Children Use and Understand. Mountain View, CA: Mayfield Publishing Company.

Ernest, P. (1999). Forms of Knowledge in Mathematics and Mathematics Education: Philosophical and Rhetorical Perspectives. Educational Studies in Mathematics, 38, 67-83.

Fielding, E.N. (1995). Teaching to the Strengths. Reading Today, 13(1), 29.

Friedman, P., & Alley, R., (1984). Learning/Teaching Styles: Applying the Principles. Theory into Practice, 23(1), 77-81.

Furoto, D.M. & Lang, M. (1982). Relationship of Self Concept Enhancement to Anxiety and Achievement in College mathematics. (ERIC Document Reproduction Service No. ED 218 088).

Fuson, K.C., & Brinko, K.T. (1985). The Comparative Effectiveness of Microcomputers and Flash Cards in the Drill and Practice of Basic Mathematics Facts. Journal for Research in Mathematics Education, 16(3), 225-232.

Fuson, K.C., Richards, J., Briars, D.J. (1982). The Acquisition and Elaboration of the Number Word Sequence. In C. Brainerd (Ed.), Children's Logical and Mathematical Cognition: Process in Cognitive Development. Berlin Heidelberg, New York: Springer-Verlag, 33-92.

Gardner, H. (1983). Frames of Mind: The Theory of Multiple Intelligences. New York: Basic Books.

Gardner, H. (1991). The Unschooled Mind: How Children Think and How Schools Should Teach. New York: Basic Books.

Gardner, H. (1993). Multiple Intelligences: The Theory in Practice. New York: Basic Books.

Gluck, D.H. (1991). Helping Students Understand Place Value. Arithmetic Teacher, 38, 10-13.

Grouwns, D.A. (ed.). (1992). Handbook of Research on Mathematics Teaching and Learning. New York: Macmillan.

Haught, L., Kunce, C., Pratt, P., Werneske, R., & Zernel, S. (2002). Improving Mastery of

Basic Mathematics Facts in Elementary School through Various Learning Strategies. (ERIC Document Reproduction Service No. ED 465 533).

Heege, H.T. (1985). The Acquisition of Basic Multiplication Skills. _Educational Studies in Mathematics, 16_(4), 375-388.

Hiebert, J., & Lindquist, M.M. (1990). Developing Mathematical Knowledge in the Young Child. In J.N. Payne (Ed.), _Mathematics for the Young Child._ Reston, VA: The National Council of Teachers of Mathematics, 17-36.

Jerman, M. (1970). Some Strategies for Solving Simple Multiplication Combinations. _Journal for Research in Mathematics Education, 1_, 95-128.

Johnson, L. (1996). The Braille Literacy Crisis for Children. _Journal of Visual Impairment & Blindness, 90_, 276-278.

Kapperman, G., Heinze, T., & Stricken, J. (1997). _Strategies for Developing Mathematics Skills in Students Who Use Braille_. Sycamore, IL: Research and Development Institute.

Kapperman, G., & Sticken, J. (1998). The Braillewriter as a Calculation Tool. _Re:View, 30_(2), 65-83.

Kapperman, G., & Sticken, J. (2002). A Software Tutorial for Learning the Nemeth Code of Braille Mathematics. _Journal of Visual Impairment & Blindness, 96_(12), 855-857.

Kapperman, G., & Sticken, Jodi. (2003). A Case for Increased Training in the Nemeth Code of Braille Mathematics for Teachers of Students Who Are Visually Impaired. _Journal of Visual Impairment & Blindness, 97_(2), 110-112.

Kelly, W.P., & Tomhave, W. (1985). A Study of Math Anxiety and Math Avoidance in Preservice Elementary Teachers. _Arithmetic Teacher, 32_, 51-53.

Kitcher, P. (1984). _The Nature of Mathematical Knowledge_. Oxford: Oxford University Press.

Kitcher, P. (1991). Persuasion. In Pera, M., & Shea, W.R. (Eds.), _Persuading Science: The Art of Scientific Rhetoric._ New York: Science History Publications, 3-27.

Kliman, M., & Russell, S.J. (1998). Building Number Sense. _Investigations in Number, Data, and Space, Grade 1_. White Plains, NY: Dale Seymour Publications.

Kline, K. (1998). Kindergarten Is More Than Counting. _Teaching Children Mathematics, 5_(2), 84-87.

Kontogianes, J.T. (1974). The Effect of Achievement, Retention, and Attitude of an Individualized Instructional Program in Mathematics for Prospective Elementary School Teachers. (Doctoral dissertation, University of Oklahoma, 1973.) _Dissertation Abstracts International, 34_, 5802-A.

LeFevre, J., Bisanz, J., Daley, K. E., Buffone, L., Greenham, S.L., & Sadesky, G. S. (1996). Multiple Routes to Solution of Single-Digit Multiplication Problems. _Journal of Experimental Psychology: General, 125_, 284-306.

LeFevre, J., Sadesky, G. S., & Bisanz, J. (1996). Selection of Procedures in Mental Addition: Reassessing the Problem Size Effect in Adults. _Journal of Experimental Psychology: Learning, Memory, and Cognition, 22_, 216-230.

Martinez, J.G.R. (1987). Preventing Math Anxiety: A Prescription. _Academic Therapy, 23_, 117-125.

Mathematical Sciences Education Board. (1996). _The Preparation of Teachers of Mathematics: Considerations and Challenges_. Washington, DC: National Research Council.

Mixon, K. (2004). Three Learning Styles...Four Steps to Reach Them. _Teaching Music, 11_(4), 48-52.

Mullen, E.A. (1990). Decreased Braille Literacy: A Symptom of a System in Need of Reassessment. _RE:view, 22_, 164-168.

Muth, D.K. (1997). Using Cooperative Learning to Improve Reading and Writing in Mathematical Problem Solving. _Reading and Writing Quarterly, 13_(1), 71-82.

Nagel, N.G., & Swingen, C.C. (1998). Students' Explanations of Place Value in Addition and Subtraction. _Teaching Children Mathematics, 5_(3), 164-170.

National Council of Teachers of Mathematics (NCTM). (1989). _Curriculum and Evaluation Standards for School Mathematics_. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). _Principles and Standards for School Mathematics_. Reston, VA: The National Council for Teachers of Mathematics, Inc.

National Research Council. (1989). _Everybody Counts_. Washington, DC: National Academy Press.

Noddings, N. (1985). Small Groups as a Setting for Research on Mathematical Problem Solving. In E.A. Silver (Ed.), _Teaching and Learning Mathematical Problem Solving_ (345-359). Hillsdale, NJ: Erlbaum.

Payne, J., & Huinker, D. (1993). Early Numbers and Numeration. In Jensen, R. (Ed.), _Research Ideas for the Classroom: Early Childhood Mathematics_. New York: Macmillan.

Peterson, P.L., & Swing, S.R. (1985). Students' Cognitions as Mediators of the Effectiveness of Small-Group Learning. _Journal of Educational Psychology, 77_, 299-312.

Piaget, J. (January 1975). Comments on Mathematical Education. _Contemporary Education, 47_(1), 5-10.

Polya, George. (1957). _How to Solve It: A New Aspect of Mathematical Method._ 2nd Edition. New York: Doubleday.

Post, T.R. (Ed.). (1992). _Teaching Mathematics in Grades K-8_. Boston: Allyn & Bacon.

Pugalee, D.K. (1995). _Using Journal Writing to Characterize Mathematical Problem Solving_. Doctoral dissertation, University of North Carolina at Chapel Hill.

Pugalee, D.K. (1997). Connecting Writing to the Mathematics Curriculum. _Mathematics Teacher, 90_(4), 308-10.

Pugalee, D.K. (1999). Constructing a Model of Mathematical Literacy. _Clearing House, 73_(1), 19-22.

Rex, E.J. (1989). Issues Related to Literacy of Legally Blind Learners. _Journal of Visual Impairment & Blindness, 83_, 306-307, 310-313.

Reys, R.E., Suydam, M.N., & Lindquist, M.N. (1995). _Helping Children Learn Mathematics._ Boston: Allyn & Bacon.

Rotman, B. (1993). _Ad Infinitum, The Ghost of Turing's Machine_. Stanford, CA: Stanford University Press.

Sarasin, L.C. (1998). _Learning Styles Perspectives: Impact in the Classroom._ Madison, Wis.: Atwood Publishing.

Sherman, H.H, Richardson, L.I, Yard, G.J. (2005). _Teaching Children Who Struggle with Mathematics: A Systematic Approach to Analysis and Correction._ Upper Saddle River, NJ: Pearson Merrill Prentice Hall.

Skemp, R.S. (1971). _The Psychology of Learning Mathematics._ Hammondsworth, England: Penguin Books.

Skemp, R. (1979). _Intelligence, Learning, and Action_. Somerset, NJ: John Wiley and Sons.

Smith, S.S. (1997). _Early Childhood Mathematics_. Boston: Allyn & Bacon.

Sovchik, R.J. (1996). _Teaching Mathematics to Children_. New York: HarperCollins.

Sovchik, R., Meconi, L.J., & Steiner, E. (1981). Mathematics Anxiety of Preservice Elementary Mathematics Methods Students. _School Science and Mathematics, 81_, 643-648.

Spungin, S.J. (1996). Braille and Beyond: Braille Literacy in a Larger Context. _Journal of Visual Impairment & Blindness, 90_, 271-274.

Spungin, S.J. (1989). _Braille Literacy: Issues for Blind Persons, Families, Professionals, and Producers of Braille_. New York: American Foundation for the Blind.

Stratton, J.M. (1996). Emergent Literacy: A New Perspective. _Journal of Visual Impairment & Blindness, 90_, 177-183.

Thornton, C.A. (1990). Strategies for the Basic Facts. In Payne, J.N. (Ed.), _Mathematics for the Young Child._ Reston, VA: National Council of Teachers of Mathematics, 132-51.

Troutman, A.P., & Lichtenberg, B.K. (1995). _Mathematics: A Good Beginning_. Pacific Grove, CA: Brooks/Cole Publishing Company.

Van de Walle, J.A. (1973). _Attitudes and Perceptions of Elementary Mathematics Possessed by Third and Sixth Grade Teachers as Related to Student Attitude and Achievement in Mathematics._

(Doctoral dissertation, Ohio State University, 1972). Dissertations Abstracts International, 33, 4254-A.

Van de Walle, J. (1990). Concepts of Number. In J.N. Payne (Ed.), *Mathematics for the Young Child.* Reston, VA: The National Council of Teachers of Mathematics, 63-87.

Vinson, B.M. (January 2001). Fishing and Vygotsky's Concept of Effective Education. *Young Children, 56*(1), 88-89.

Vinson, B.M. (Winter 2001). A Comparison of Preservice Teachers' Mathematics Anxiety Before and After a Methods Class Emphasizing Manipulatives. *Early Childhood Education Journal, 29*(2), 89-94.

Vygotsky, L.S. (1962). *Thought and Language*. New York: MIT Press and Wiley.

Vygotsky, L.S. (1967). Play and Its Role in the Mental Development of the Child. *Soviet Psychology, 5*(3), pp. 6-18.

Webb, N.M. (1984). Stability of Small-Group Interaction and Achievement Over Time. *Journal of Educational Psychology, 76*, 211-224.

Webb, N.M. (1985). Verbal Interaction and Learning in Peer-Directed Groups. *Theory into Practice, 24*, 32-39.

Webb, N.M. (1991). Task-Related Verbal Interaction and Mathematics Learning in Small Groups. *Journal for Research in Mathematics Education, 22*, 366-389.

Whitenack, J., Knipping, N., & Underwood, G. (2001). Second Graders Circumvent Addition and Subtraction Difficulties. *Teaching Children Mathematics, 8*(4), 228-233.

Willis, J.K. (2001). Multiply with MI: Using Multiple Intelligences to Master Multiplication. *Teaching Children Mathematics, 7*(5), 260-268.

Wirtz, R. (1980). *New Beginnings. A Guide to the Think, Talk, Read Math Center for Beginners*. Monterey, CA: Curriculum Development Associates.

Wittenstein, S.H. (1994). Braille Literacy: Preservice Training and Teachers' Attitudes. *Journal of Visual Impairment & Blindness, 88*, 516-524.

Wittenstein, S.H., & Pardee, M.L. (1996). Teachers' Voices: Comments on Braille and Literacy from the Field. *Journal of Visual Impairment & Blindness, 90*, 201-209.

Wright, D.E., & Miller, L.D. (1981). Math Anxiety: A Research Report. (ERIC Document Reproduction Service No. ED 212 465, 97 pages.)