

# TOUCHMATH<sup>®</sup>

THE ALPHABET OF MATHEMATICS

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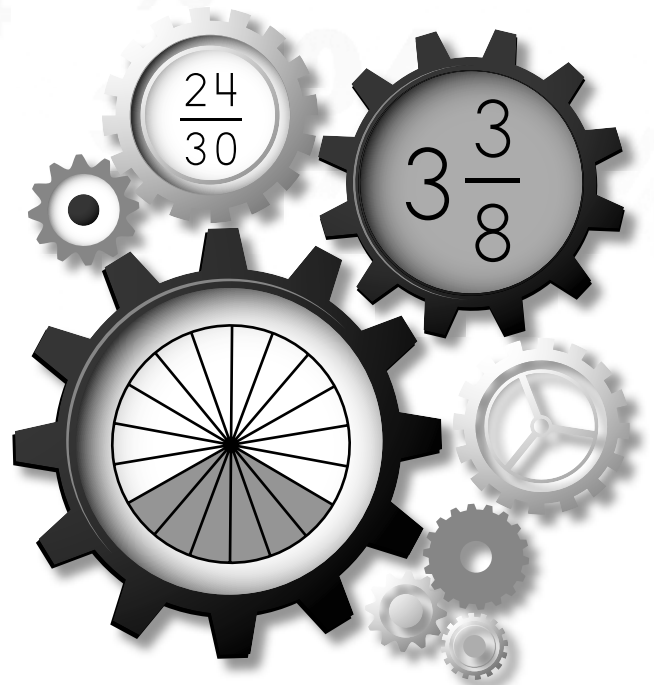
UNIT  
8

## CLASSIC UPPER GRADES

### FRACTIONS, DECIMALS, & PERCENTS 2

#### MODULE TITLES

- 1: Addition and Subtraction of Fractions
- 2: Multiplication and Division of Fractions
- 3: Addition and Subtraction of Decimals
- 4: Multiplication and Division of Decimals
- 5: Percents
- 6: Mixed Practice
- 7: Application
- 8: Mixed Review



# ► Fractions, Decimals, and Percents 2

## Unit 8 ◉

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# ► Fractions, Decimals, and Percents 2

## Unit 8 ○

## Introduction

### Organization

The following paragraphs describe the structure of the curriculum. If you would like more information about TouchMath, our teacher training DVD is available at no charge. Request online at [www.touchmath.com/freetraining](http://www.touchmath.com/freetraining), or call 1-800-888-9191.

### Unit Components

The goals for each unit are defined in the overview of skills. These broad proficiencies often establish the framework for concepts of increasing complexity. The goals are broken down into clear, manageable objectives that list the academic expectations of the students and summarize the module-level objectives. Unit vocabulary and detailed Common Core State Standards complete the unit overview. The unit pre- and post-tests immediately follow with directions for administering, recording results, and using the results to determine each student's educational plan.

### Module Guides

The table of contents provides the skeleton of the activities within each module guide. The modules include clusters—subsets of the featured skill. A paragraph overview of the module

- identifies the clusters,
- explains the activities,
- lists the Common Core State Standards by their code,
- specifies objectives in the order of presentation,
- labels basic prerequisites,
- lists vocabulary necessary for skill attainment, and
- suggests readily available materials that would be helpful during the lesson

The lessons in the modules begin with a pretest, which gives basic directions for completion. It is recommended that you give little instruction related to the skill before testing. A record sheet is included for tracking student achievement. This record is found on the third page of each module guide. Instructional strategies follow the pretest, providing ideas for the most effective use of the student activity sheets. Four different formatting conventions reveal which type of strategy is being offered:

**Box**: Information in this shape is background information for the teacher, explaining the skill and illuminating the purpose and/or value of mastery.

☞: A speech bubble offers what the teacher is to say to the class. Anyone presenting the lesson could use this script.

◆: A diamond bullet suggests action for the teacher. It typically includes directions such as "Write ... on the whiteboard." "Monitor students as they complete the row of problems."

**Bold**: Directions in bold type suggest actions relating to transitions. These include statements such as "Distribute activity sheets ... to the students." "Activity Sheet ... Directions." "Repeat the activity sheet xx process ..."

The answer keys are imbedded in the instructional strategies for a quick reference while planning or presenting the lesson. Modified directions for the activity sheets are included for use after the detailed, step-by-step process to ensure understanding of the concepts.

A posttest follows the instruction within the module. Refer to the module guide for directions for administering the posttest. You can record results and compare them to the pretest. The module concludes with suggestions for differentiated instruction and real world applications.

# ► Addition and Subtraction of Fractions

Distribute activity sheet 15 to the students.

**Instruction: activity sheet 15**

- ☞ Please look at this activity sheet, and think about how it is different from the previous ones we used for addition and subtraction.

Yes, the equations are written under one another instead of beside each other. The equal symbol is in a vertical line, as we would line up numbers to add or subtract them. You will see this format more often as you advance through math.<sup>1</sup>

The first example is  $1/3 + 1/4$ . The solid lines show the division into equal parts for  $1/3$  and  $1/4$ . The shaded diagonal lines show  $1/3$  on the first model, the first added. That amount is shown the same way on the model with the sum.

Look at the equations below the models. We see that the first equation,  $1/3 + 1/4$ , reflects each model. The equation below it shows the fractions with a **common denominator**. The **common denominator** for thirds and fourths is twelfths. Can you see that  $1/3 = 4/12$ ? Can you see that  $1/4 = 3/12$ ? The dotted lines within each circle show the parts with the **common denominator**. Do you see that each model is divided into the same number of parts by the dotted lines? When we add  $3/12$  and  $4/12$ , we add the numerators. The sum is  $7/12$ . Do you see the  $7/12$  shaded on the model for the sum?  $4/12$  is shaded with diagonal lines, and  $3/12$  is shaded gray. The total shaded fractional part is  $7/12$ .

Work with your partner to do the second problem. Use your crayons for shading, or cut out each model if you need to. Talk about each step.

Let's check your work. Did you and your partner write  $1/2 + 2/3 = ?$  Did you write  $3/6 + 4/6 = 7/6 = 1 1/6$ ?

Complete problem 3 on your own. Shade the circles for the sum to show the addition.

What can you say about the **common denominator** for each of the examples? Think about it. Discuss it with your partner. Yes, the **common denominator** in each of the examples is the product of the denominators of the two addends.

**Independent Practice**

- ☞ Use the models. Write the missing numbers in the equations. Shade the model for the sum in problem 3.

Distribute activity sheet 16 to the students.

**Instruction: activity sheet 16**

- ☞ On this activity sheet, we will repeat the processes used on the previous activity sheets to find differences of fractions. Let's review what we know about the models used on this sheet.

There are two kinds of lines and two kinds of shading. The solid lines within the shape show the number of parts with the smaller denominator. The dotted lines show the larger denominator, the **common denominator**.

Activity sheet 15 contains three problems. Each problem shows a circular model divided into equal parts by solid lines, with a larger model divided into smaller parts by dotted lines. The equations are written below the models, showing the addition of two fractions and the result with a common denominator.

1.  $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$   
 $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

2.  $\frac{1}{2} + \frac{2}{3} = \frac{7}{6} = 1\frac{1}{6}$   
 $\frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$

3.  $\frac{1}{2} + \frac{4}{5} = \frac{9}{10} = 1\frac{3}{10}$   
 $\frac{5}{10} + \frac{8}{10} = \frac{13}{10} = 1\frac{3}{10}$

15

**Basic Background**

<sup>1</sup>This presentation is a soft introduction to rewriting equations in pre-algebra.

Activity sheet 16 contains five problems. Problems 1 and 2 use rectangular models, while problems 3, 4, and 5 use circular models. Each problem shows the subtraction of two fractions and the result with a common denominator.

1.  $\frac{1}{2} - \frac{2}{7} = \frac{3}{14}$   
 $\frac{7}{14} - \frac{4}{14} = \frac{3}{14}$

2.  $\frac{3}{5} - \frac{1}{3} = \frac{4}{15}$   
 $\frac{9}{15} - \frac{5}{15} = \frac{4}{15}$

3.  $\frac{1}{4} - \frac{2}{3} = \frac{-5}{12}$   
 $\frac{3}{12} - \frac{8}{12} = \frac{-5}{12}$

4.  $\frac{1}{2} - \frac{4}{5} = \frac{-3}{10}$   
 $\frac{5}{10} - \frac{8}{10} = \frac{-3}{10}$

5.  $\frac{2}{3} - \frac{16}{24} = \frac{-10}{24}$   
 $\frac{-5}{8} - \frac{15}{24} = \frac{-20}{24}$

6.  $\frac{4}{9} - \frac{12}{27} = \frac{0}{27} = 0$   
 $\frac{-1}{3} - \frac{9}{27} = \frac{-10}{27}$

16

- The gray shading is the total amount of the first fraction. The shading with diagonal marks is the amount that is taken away, separated from, the total gray shading. The remaining shaded area is the difference.

See if you can do the first problem on your own. Read the equations. Draw a ring around the part that remains after a part is separated. Say and write the missing number.

Complete the second problem.

Look at the problem 3. How is it different from the ones you just completed? Yes, a fractional part is subtracted from a mixed number. Notice the lines inside each circle. The solid lines show the fractional part of the mixed number. The **bold** lines show the fractional part of the fraction to be subtracted. The dotted lines show the **common denominator**. Think about this, and discuss it with your partner. The model is created so you can see what happens when you subtract a fraction from a mixed number.

Please use your yellow crayon to shade  $1 \frac{1}{4}$  of the circles. Use the solid lines within the circle as a guide. Use your blue crayon to shade the  $\frac{2}{3}$  that has diagonal marks, the part to be subtracted or taken away.

Count the number of parts that remain, the parts still shaded yellow. Yes,  $\frac{7}{12}$ .

It can be difficult to see the amount that remains when we use these **visual** models. We have to take the  $\frac{2}{3}$  from the whole because there are not enough parts to take it from the fraction of the mixed number.

Read the equations with me:  $1 \frac{1}{4} - \frac{2}{3} = \frac{5}{4} - \frac{2}{3} = \frac{15}{12} - \frac{8}{12} = \frac{7}{12}$ .

Talk with your partner about  $\frac{5}{4} = \frac{15}{12}$ . Explain how that is true. Use the model if it helps you.<sup>1</sup>

Stand up if you think you can complete the fourth problem on your own.<sup>2</sup>

Look at problems 5 and 6. You will solve these equations without models. In the fifth problem, how do you know that the **common denominator** is 24? Yes, because it is the **least common multiple** of 3 and 8. Working with your partner, write the multiples of 3 in one column and the multiples of 8 in another column on the back of this sheet. Skip count to get the multiples. List multiples until you have a common multiple.

What is an easier way to find the **least common multiple**? Yes, find the product of the two denominators. Write the equivalent fraction for  $\frac{2}{3}$  that has a denominator of 24. Do the same for  $\frac{5}{8}$ . Subtract. Write the difference. Say the equation.

Complete the sixth problem on your own.

### Independent Practice

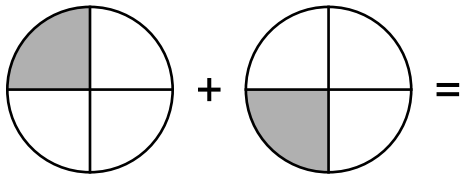
- Use the models. Subtract. Find the missing numbers. Complete the equations. Simplify.

### Instructional Insight

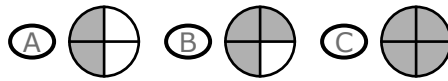
<sup>1</sup>Using a visual model to introduce subtraction of a fraction from a mixed number is a soft introduction to regrouping. The goal is for students to see and understand that when a whole number of a mixed number is regrouped, it is regrouped into the number of fractional parts of the fraction. Students often transfer learning incorrectly and think that they regroup 10 parts, as they did with whole numbers.

### Differentiated Directions

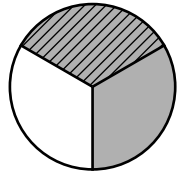
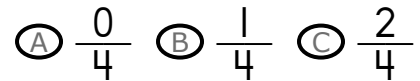
<sup>2</sup>Continue to provide guided practice and coach students as they manipulate the models that represent subtraction of fractions.



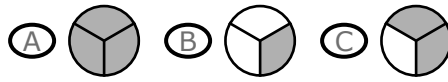
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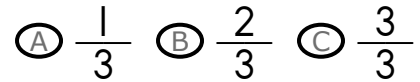
2.



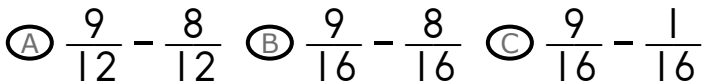
3.



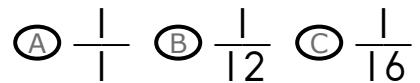
4.



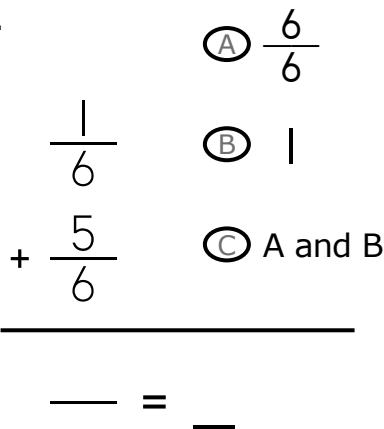
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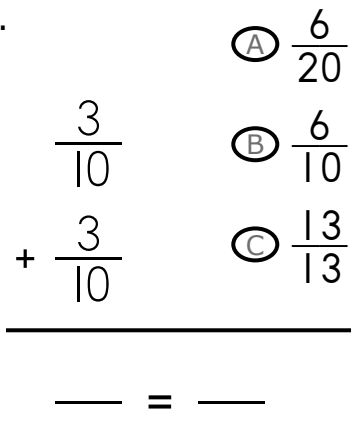
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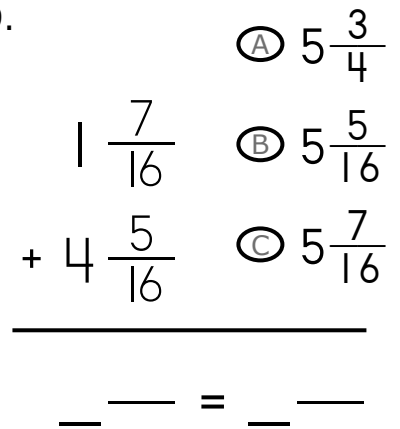
7.



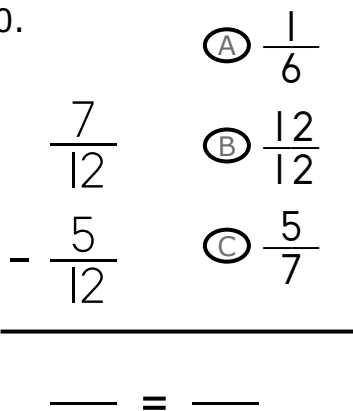
8.



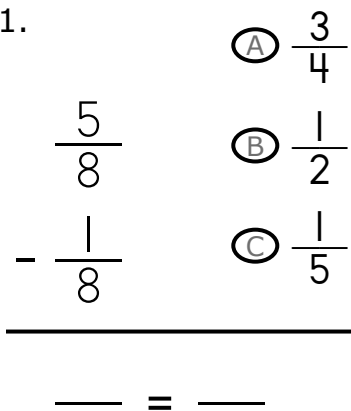
9.



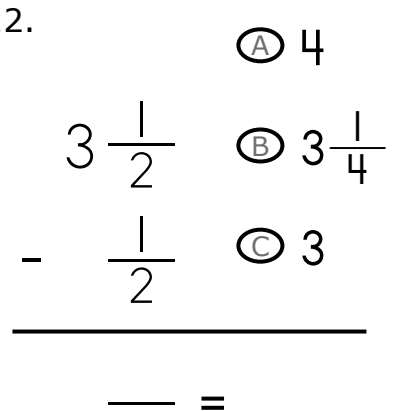
10.



11.



12.





=

13.



14.

- (A)  $\frac{4}{12}$  (B)  $\frac{4}{8}$  (C)  $\frac{7}{8}$



=

15.

- (A)  $\frac{1}{3} + \frac{5}{12}$  (B)  $\frac{1}{6} + \frac{2}{3}$  (C)  $\frac{1}{12} + \frac{5}{6}$

16.

- (A)  $\frac{4}{6}$  (B)  $\frac{6}{15}$  (C)  $\frac{9}{12}$

17.

$$\begin{array}{r} \frac{5}{8} \\ + \frac{3}{5} \\ \hline \end{array}$$

(A)  $\frac{8}{13}$   
 (B)  $\frac{39}{40}$   
 (C)  $1\frac{9}{40}$

\_\_\_\_\_ = \_\_\_\_\_

18.

$$\begin{array}{r} 1\frac{1}{6} \\ + 1\frac{2}{9} \\ \hline \end{array}$$

(A)  $2\frac{1}{5}$   
 (B)  $2\frac{7}{18}$   
 (C)  $2\frac{3}{54}$

\_\_\_\_\_

19.

$$\begin{array}{r} 1\frac{11}{16} \\ - \frac{5}{8} \\ \hline \end{array}$$

(A)  $\frac{1}{8}$   
 (B)  $\frac{1}{16}$   
 (C)  $\frac{6}{16}$

\_\_\_\_\_

20.

$$\begin{array}{r} 1\frac{3}{8} \\ - \frac{5}{6} \\ \hline \end{array}$$

(A)  $\frac{1}{2}$   
 (B)  $1\frac{2}{3}$   
 (C)  $\frac{13}{24}$

\_\_\_\_\_

21.

$$\begin{array}{r} 3\frac{1}{8} \\ - 1\frac{1}{2} \\ \hline \end{array}$$

(A)  $1\frac{5}{8}$   
 (B)  $2\frac{0}{6}$   
 (C)  $1\frac{7}{8}$

\_\_\_\_\_

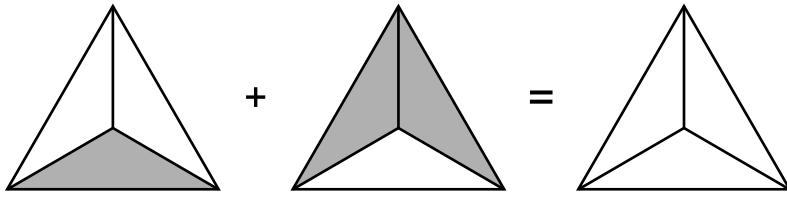
22.

$$\begin{array}{r} 8\frac{2}{3} \\ + 6\frac{3}{4} \\ \hline \end{array}$$

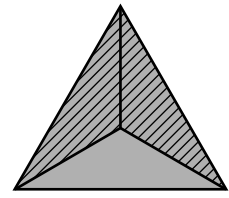
(A)  $14\frac{5}{7}$   
 (B)  $15\frac{5}{12}$   
 (C)  $15\frac{7}{12}$

\_\_\_\_\_ = \_\_\_\_\_

1.

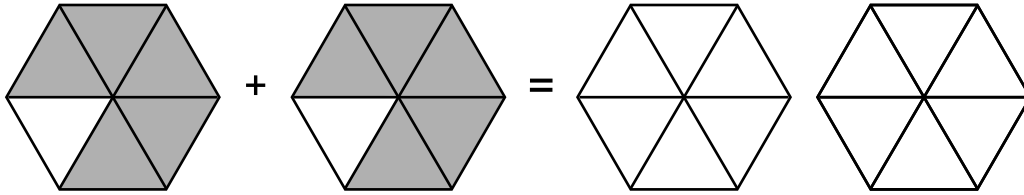


$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$



$$\frac{2}{3} = \frac{2}{3}$$

2.



$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

3.



$$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

4.



$$\frac{5}{10} + \frac{5}{10} = \frac{10}{10}$$

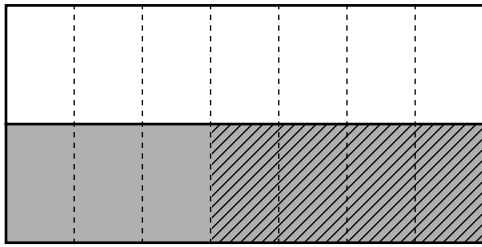
5.



$$\frac{4}{6} + \frac{2}{6} = \frac{6}{6} = 1$$



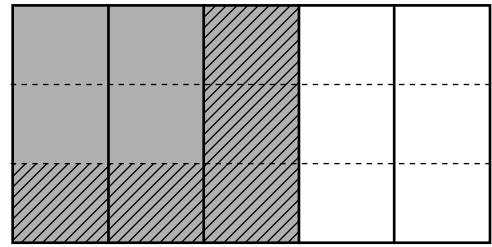
1.



$$\frac{1}{2} - \frac{2}{7} =$$

$$\frac{7}{14} - \frac{4}{14} = \frac{\quad}{14}$$

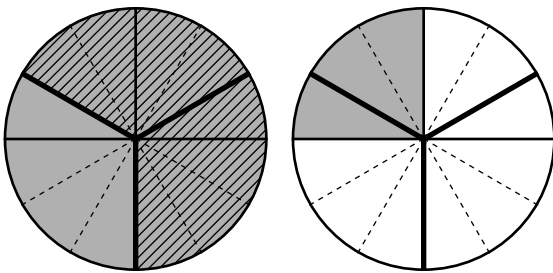
2.



$$\frac{3}{5} - \frac{1}{3} =$$

$$\frac{\quad}{15} - \frac{5}{15} = \frac{4}{\quad}$$

3.

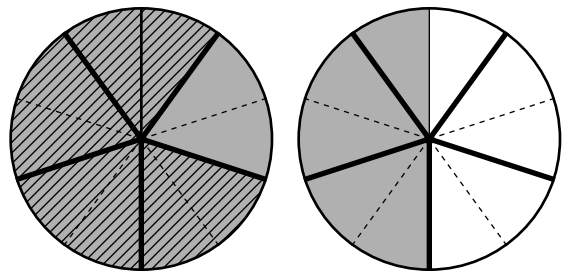


$$1 \frac{1}{4} - \frac{2}{3} =$$

$$\frac{5}{4} - \frac{2}{3} =$$

$$\frac{15}{12} - \frac{8}{12} = \frac{\quad}{12}$$

4.



$$1 \frac{1}{2} - \frac{4}{5} =$$

$$\frac{3}{2} - \frac{4}{5} =$$

$$\frac{\quad}{10} - \frac{8}{10} = \frac{\quad}{\quad}$$

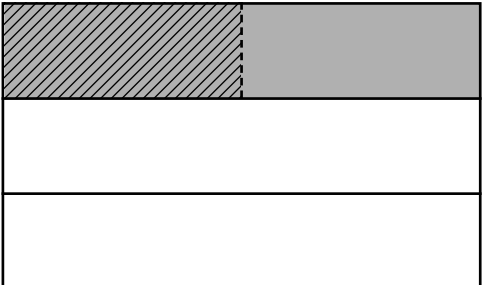
5.

$$\begin{array}{r} \frac{2}{3} = \frac{\quad}{24} \\ - \frac{5}{8} = \frac{\quad}{24} \\ \hline \frac{\quad}{24} \end{array}$$

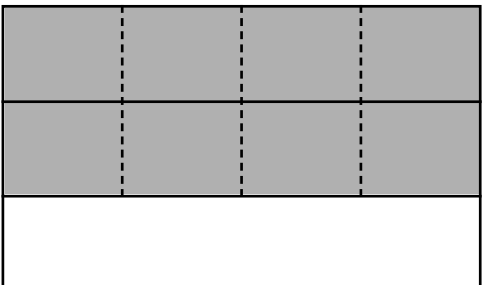
6.

$$\begin{array}{r} \frac{4}{9} = \frac{12}{\quad} \\ - \frac{1}{3} = \frac{9}{\quad} \\ \hline \frac{\quad}{\quad} = \frac{\quad}{\quad} \end{array}$$


1.

$\frac{1}{3} \div 2 =$ 

 $= \frac{1}{3} \div \frac{2}{1}$   
 $= \text{---} \times \text{---} = \text{---}$

2.

$\frac{2}{3} \div 4 =$ 

 $= \frac{2}{3} \div \text{---}$   
 $= \frac{2}{3} \times \text{---} = \text{---}$

3.

$\frac{5}{8} \div 2 =$ 

 $= \text{---} \div \text{---}$   
 $= \text{---} \times \text{---} = \text{---}$

To divide a fraction  
by a whole number:

1. Write the first fraction.
2. Rename the divisor as a fraction.
3. Invert the divisor.
4. Multiply the numerators.
5. Multiply the denominators.

1.

$$\frac{1}{2} \times \frac{4}{5} = \begin{array}{|c|c|c|c|c|} \hline \text{shaded} & \text{shaded} & \text{shaded} & \text{shaded} & \text{white} \\ \hline \end{array} = \frac{1}{2} \times \frac{4}{5} = \frac{4}{10} = \frac{2}{5}$$

2.

$$\frac{1}{2} \times \frac{6}{7} = \begin{array}{|c|c|c|c|c|c|c|} \hline \text{shaded} & \text{shaded} & \text{shaded} & \text{shaded} & \text{shaded} & \text{shaded} & \text{white} \\ \hline \end{array} = \text{---} \times \text{---} = \text{---} = \text{---}$$

3.

$$\frac{1}{2} \times \frac{2}{3} = \begin{array}{|c|c|c|} \hline \text{shaded} & \text{shaded} & \text{white} \\ \hline \end{array} = \text{---} \times \text{---} = \text{---} = \text{---}$$

4.

$$\frac{1}{3} \times \frac{9}{10} = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline \text{white} & \text{white} & \text{white} & \text{white} & \text{white} & \text{white} & \text{white} & \text{white} & \text{white} & \text{white} \\ \hline \end{array} = \text{---} \times \text{---} = \text{---} = \text{---}$$

5.

$$\frac{3}{4} \times \frac{1}{6} = \text{---} = \text{---}$$

6.

$$\frac{4}{5} \times \frac{2}{7} = \text{---}$$

7.

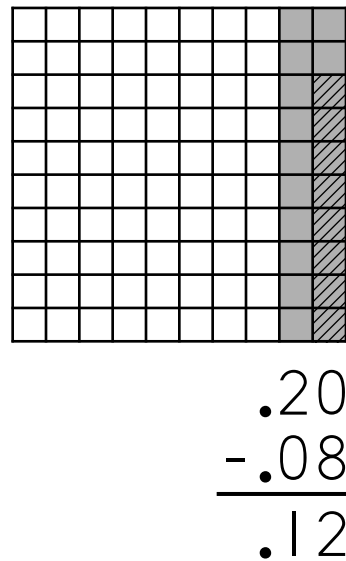
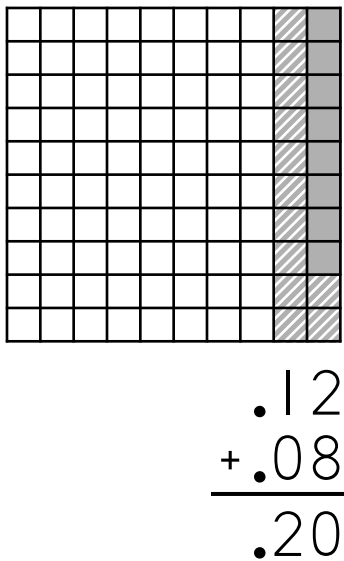
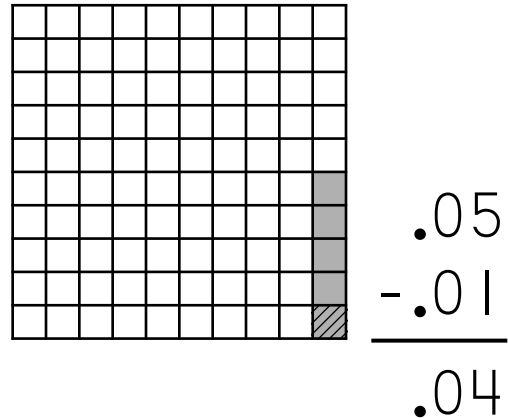
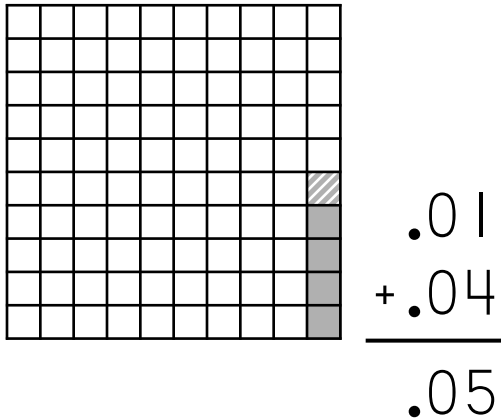
$$\frac{5}{8} \times \frac{2}{3} = \text{---} = \text{---}$$

8.

$$\frac{7}{10} \times \frac{1}{8} = \text{---}$$

To multiply a fraction by a fraction:

1. Multiply the numerators.
2. Multiply the denominators.
3. Simplify the answer.



1. 
$$\begin{array}{r} .13 \\ +.04 \\ \hline \end{array}$$

2. 
$$\begin{array}{r} .53 \\ +.22 \\ \hline \end{array}$$

3. 
$$\begin{array}{r} .87 \\ - .06 \\ \hline \end{array}$$

4. 
$$\begin{array}{r} .41 \\ - .21 \\ \hline \end{array}$$

5. 
$$\begin{array}{r} .34 \\ +.07 \\ \hline \end{array}$$

6. 
$$\begin{array}{r} .57 \\ +.26 \\ \hline \end{array}$$

7. 
$$\begin{array}{r} .91 \\ - .08 \\ \hline \end{array}$$

8. 
$$\begin{array}{r} .83 \\ - .16 \\ \hline \end{array}$$