TOUCHMATH®

THE ALPHABET OF MATHEMATICS

Since 1975

UNIT 10

CLASSIC UPPER GRADES

PRE-ALGEBRA

MODULE TITLES

1: Operations

2: Ratio

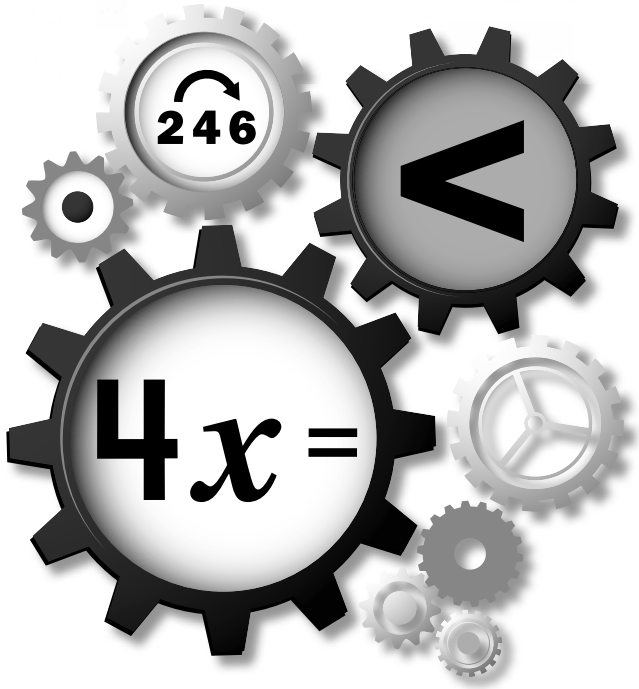
3: Rational Numbers

4: Expressions

5: Variables

6: Patterns

7: Geometry



Unit 10 ◎ **Table of Contents**

Unit 10 ◎ Introduction

Organization

The following paragraphs describe the structure of the curriculum. If you would like more information about TouchMath, our teacher training DVD is available at no charge. Request online at www.touchmath.com/freetraining, or call 1-800-888-9191.

Unit Components

The goals for each unit are defined in the overview of skills. These broad proficiencies often establish the framework for concepts of increasing complexity. The goals are broken down into clear, manageable objectives that list the academic expectations of the students and summarize the module-level objectives. Unit vocabulary and detailed Common Core State Standards complete the unit overview. The unit pre- and post-tests immediately follow with directions for administering, recording results, and using the results to determine each student's educational plan.

Module Guides

The table of contents provides the skeleton of the activities within each module guide. The modules include clusters—subsets of the featured skill. A paragraph overview of the module

- identifies the clusters,
- explains the activities,
- lists the Common Core State Standards by their code,
- specifies objectives in the order of presentation,
- labels basic prerequisites,
- lists vocabulary necessary for skill attainment, and
- suggests readily available materials that would be helpful during the lesson

The lessons in the modules begin with a pretest, which gives basic directions for completion. It is recommended that you give little instruction related to the skill before testing. A record sheet is included for tracking student achievement. This record is found on the third page of each module guide. Instructional strategies follow the pretest, providing ideas for the most effective use of the student activity sheets. Four different formatting conventions reveal which type of strategy is being offered:

Box: Information in this shape is background information for the teacher, explaining the skill and illuminating the purpose and/or value of mastery.

: A speech bubble offers what the teacher is to say to the class. Anyone presenting the lesson could use this script.

◆: A diamond bullet suggests action for the teacher. It typically includes directions such as "Write ... on the whiteboard." "Monitor students as they complete the row of problems."

Bold: Directions in bold type suggest actions relating to transitions. These include statements such as "Distribute activity sheets ... to the students." "Activity Sheet ... Directions." "Repeat the activity sheet xx process ..."

The answer keys are imbedded in the instructional strategies for a quick reference while planning or presenting the lesson. Modified directions for the activity sheets are included for use after the detailed, step-by-step process to ensure understanding of the concepts.

A posttest follows the instruction within the module. Refer to the module guide for directions for administering the posttest. You can record results and compare them to the pretest. The module concludes with suggestions for differentiated instruction and real world applications.

Cluster 2 Introduction

This cluster introduces exponents through a familiar experience: finding area and volume. Activities include work with vocabulary, the relationship between exponents and multiplication, and powers of 10 and place value. Tables, unknowns, and comparisons ensure firm concept development.

A multiple choice review precedes the module posttest.

- You learned a lot of new information about operations, and you performed operations based on this new information. Can you think of someone who also performs operations? Yes, **surgeons**. When **surgeons** perform operations, they sometimes take apart parts of the body and replace those parts with new parts. The new parts are equal to the old parts. For example, a **surgeon** replaces a lung with another lung, not with a heart. They do this to make the patient's life better.

  The same is true of operations with numbers. When you perform operations with numbers, you replace parts like 9+4 with equal parts like 13 to make the **expression** better, easier to understand. People who use algebra are similar to **surgeons**. They break complex things into smaller parts and make the parts easier to use.

  You learned about *Please My Dear Aunt Sally*. Sometimes G is used in place of P in the order of operations. The G is for *grouping*. We have used *grouping* as the key word for the commutative **property**. If you can use *grouping* as keys for understanding both a **property** and the order of operations, change the P in the phrase to G. Think of a meaningful word that starts with G. Maybe *Good* or *Gee* or *Golly*. The reason why some people use a G instead of a P is because numbers in **expressions** can be grouped with symbols other than **parentheses**.

- Demonstrate examples of other grouping symbols: ( ), [ ], { }. Explain the use of different symbols. Parentheses are used to group things inside brackets; brackets are used to group things inside braces.[1]

Distribute activity sheet 17 to the students.

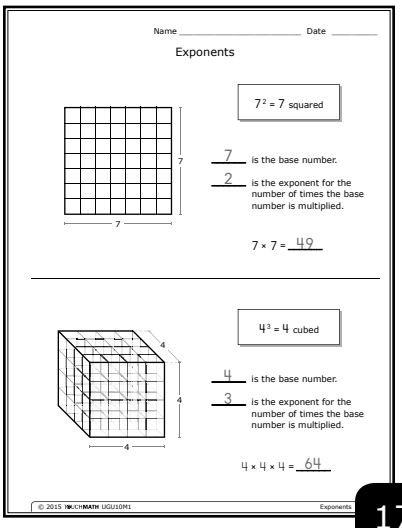
Instruction: activity sheet 17

- We will continue to explore operations, this time with **exponents**. **Exponents** involve operations in various ways.

  Does this activity sheet look familiar to you? These are the models you used to find the area of 2-D shapes and the volume of 3-D shapes. Do you remember that area is expressed in square units? Sometimes we wrote *square units*, *sq. units*, *units* ***squared***, or *units*^{2}, but they all meant the same thing. We sometimes the unit was square inches, feet or yards; square centimeters, meters, or kilometers. The area model on this sheet doesn't have a label. You can label the units however you wish.

Basic Background

[1]Different symbols are needed when the groupings are complex. Many students will not encounter the nested grouping these other symbols require.


Name _____ Date _____

Exponents

7^2 = 7 squared

7 is the base number.

2 is the exponent for the number of times the base number is multiplied.

7 × 7 = 49

4^3 = 4 cubed

4 is the base number.

3 is the exponent for the number of times the base number is multiplied.

4 × 4 × 4 = 64

 Exponents

17

- The area of this figure is 7 x 7. We could also write this as 7 **squared** or 7^2. The small 2 after the 7 is the **exponent**, and it means ***squared***. The 7 is the **base** number. It is the number that forms the basis for the **exponent**. The **exponent** 2 is extra information about the **base** number 7. Please trace the numbers and read the sentences below the box that has 7^2 = 7 **squared**.

  The **expression** 7^2 means to multiply the **base** number 7 by the **base** number 7. We do not multiply the **base** number 7 by the **exponent** 2. That would be written 7 x 2.

  Write the product of 7 x 7 on the line.

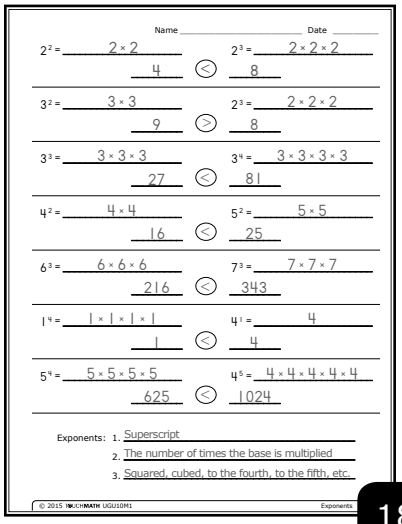
  When we found the volume of 3-D shapes, the label was written with a 3 as the **exponent**, such as cm^3. In the example at the bottom of the activity sheet, each dimension is 4 units. To find the volume, the number of cubes that are contained inside the shape, we multiply 4 x 4 x 4. With an **exponent**, this would be 4^3, 4 **cubed**. What is the **base** number? Yes, 4. It is the basis; it is the measurement of each side. How many times is 4 multiplied together? Yes, 3 times. The **exponent** is 3. It is something extra about the **base** number.

  Write the numbers on the lines to complete the sentences. Find 4 x 4 x 4, and write the solution on the line.

Independent Practice

No independent practice is recommended.

Distribute activity sheet 18 to the students.


Name _____ Date _____

2^2 = 2 × 2 = 4 < 2^3 = 2 × 2 × 2 = 8

3^2 = 3 × 3 = 9 > 2^3 = 2 × 2 × 2 = 8

3^3 = 3 × 3 × 3 = 27 < 3^4 = 3 × 3 × 3 × 3 = 81

4^2 = 4 × 4 = 16 < 5^2 = 5 × 5 = 25

6^3 = 6 × 6 × 6 = 216 < 7^3 = 7 × 7 × 7 = 343

1^4 = 1 × 1 × 1 × 1 = 1 < 4^1 = 4 = 4

5^4 = 5 × 5 × 5 × 5 = 625 < 4^5 = 4 × 4 × 4 × 4 × 4 = 1024

Exponents: 1. Superscript
2. The number of times the base is multiplied
3. Squared, cubed, to the fourth, to the fifth, etc.

 Exponents

18

Instruction: activity sheet 18

- On this activity sheet, you will practice using **exponents**. When you have completed the practice, we will work together to form some generalizations about **exponents**.

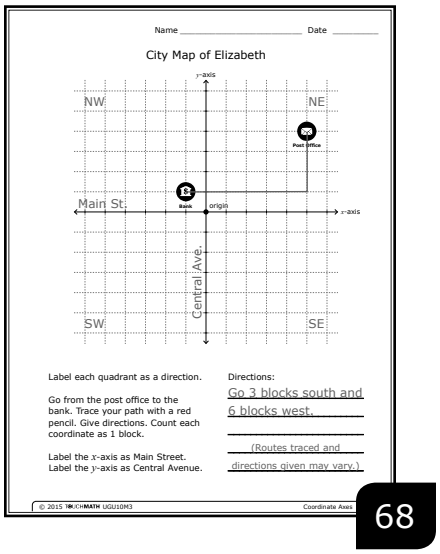
  Let's do the first and last rows together. 2^2 = 2 x 2. It means the **base** number is written twice. The product is 4. Trace the numbers. 2^3 = 2 x 2 x 2. The 2 is the factor that is multiplied. The 3 is the number of times the factor is multiplied. Trace the numbers and the value of 2^3.

  Now compare the values of 2^2 and 2^3. Trace the symbol for the comparison.

  Think about the comparison. What generalization can you make based on that example? Discuss it with your partner. In this case, the **base** number is the same for both examples, but the **exponent** is different.

  Now let's do the last problem. When the **exponent** is greater than 3, we use the fractional name of the **exponent**. In this problem, we say *five to the fourth power* and *four to the fifth power*. Although they are said differently than **exponents** of 2 or 3, they still indicate how many times the **base** number must be multiplied by itself.

Distribute activity sheet 68 to the students.


Name _____ Date _____

City Map of Elizabeth

Label each quadrant as a direction.

Go from the post office to the bank. Trace your path with a red pencil. Give directions. Count each coordinate as 1 block.

Label the x-axis as Main Street.
Label the y-axis as Central Avenue.

Directions:
Go 3 blocks south and 6 blocks west.
(Routes traced and directions given may vary.)

 Coordinate Axes

68

Instruction: activity sheet 68

- We will do this activity sheet together. Read the first line of directions. We are to label the direction of each **quadrant**. Let's label the **axes** first. That will make it easier to figure out the direction of the **quadrants**. Since this is a map of a town, let's say that up is north and down is south. Write N at the top of the y-axis and S at the bottom of it. Let's say that left is west and right is east. Write E at the right end of the x-axis and W at the left end of it.

  Now we can label the **quadrants**. **Quadrant** I is between what two directions? Yes, north and east. With your pencil, write NE inside the **quadrant** for northeast. Make it light so that it does not interfere with anything else.

  Can you label the direction of each of the other **quadrants**? The direction of the **quadrant** is a combination of the two directions that border it. **Quadrant** II is NW, **quadrant** III is SW, and **quadrant** IV is SE.

  Locations in towns and cities are sometimes referred to the directional **quadrant**, just as we have labeled here. This gives people a reference point for finding a particular location.

  Read the next set of instructions. Starting at the post office, follow the dotted lines for the streets. Do not move diagonally. There are two direct paths from the post office to the bank.

  Let's move parallel to the y-axis first. Do we need to move north or south to get from the post office to the bank? Yes, south. How many blocks do we need to go? Yes, 3. Trace this path with your red pencil. In the space for the directions, write *Go 3 blocks south*.

  Now what direction will we travel? Yes, west. How far west? Yes, 6 blocks. Trace this path with your red pencil, and write *Go 6 blocks* west in the space for the directions.[1]

  Read the next two statements, and follow the instructions. Describe to your partner another way to get from the post office to the bank, this time using the street names.[2]

  Coordinate planes are also grids. Many plans are built on grids. Graphic designers use grids for placing information on paper. City and traffic planners use grids of their community for locating the municipal building, utility buildings, stop lights, etc. Maps of cities and states are on scaled grids. Architects use grids to draw homes on a scale so the homeowner can see the relative sizes of the rooms. The power structure for the United States is on a number of grids. When too much power is used, the grid can shut down. Grids are models that have many uses. Understanding how information is identified on a grid is the same as understanding how points are located on a coordinate plane.

  Using this map of Elizabeth, work with your partner to locate another building. Write instructions to get to that building from the post office and from the bank.

Instructional Insight

[1]Giving directions to get from one location to another is a realistic application. This is also a soft introduction to absolute value since one cannot walk a negative number of blocks.

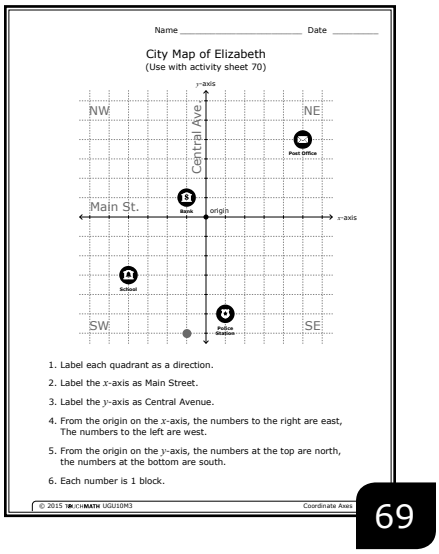
Instructional Insight

[2]Giving directions with street names is often helpful to the receiver of the information.

Independent Practice

No independent practice is recommended.

Repeat the activity sheet 68 process for sheets 69–70, or assign the sheets to be completed independently.



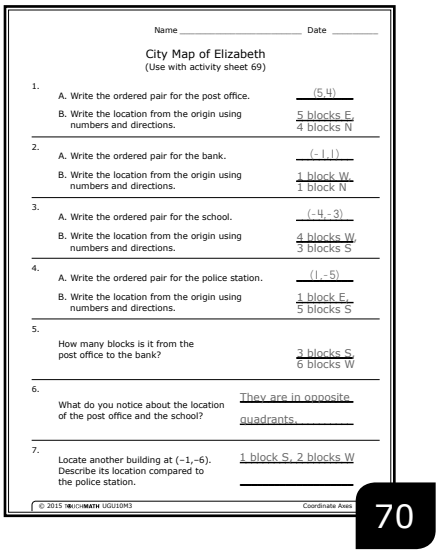
Name _____ Date _____

City Map of Elizabeth
(Use with activity sheet 70)

1. Label each quadrant as a direction.
2. Label the x-axis as Main Street.
3. Label the y-axis as Central Avenue.
4. From the origin on the x-axis, the numbers to the right are east, The numbers to the left are west.
5. From the origin on the y-axis, the numbers at the top are north, the numbers at the bottom are south.
6. Each number is 1 block.

 Coordinate Axes

69



Name _____ Date _____

City Map of Elizabeth
(Use with activity sheet 69)

1.
A. Write the ordered pair for the post office. (5,4)
B. Write the location from the origin using numbers and directions. 5 blocks E, 4 blocks N

2.
A. Write the ordered pair for the bank. (-1,1)
B. Write the location from the origin using numbers and directions. 1 block W, 1 block N

3.
A. Write the ordered pair for the school. (-4,-3)
B. Write the location from the origin using numbers and directions. 4 blocks W, 3 blocks S

4.
A. Write the ordered pair for the police station. (1,-5)
B. Write the location from the origin using numbers and directions. 1 block E, 5 blocks S

5.
How many blocks is it from the post office to the bank? 3 blocks S, 6 blocks W

6.
What do you notice about the location of the post office and the school? They are in opposite quadrants.

7.
Locate another building at (−1,−6). Describe its location compared to the police station. 1 block S, 2 blocks W

 Coordinate Axes

70

Independent Practice

- Read and follow the instructions on activity sheet 69 to complete the map of Elizabeth. Use the map to complete activity sheet 70.

Name _____ Date _____

1. The identity property that uses 1 applies to _____.

(A) × and ÷
(B) + and −
(C) − and ÷
(D) none of the above

2. In the order of operations, if there are no parentheses, _____ first.

(A) + or −
(B) × or ÷
(C) both A and B
(D) none of the above

3. The value of $6 \times (7 + 8)$ is _____.

(A) 42 + 48
(B) 6 × 15
(C) 90
(D) all of the above

4. In 5^2, the 2 is the _____.

(A) base number
(B) order
(C) exponent
(D) none of the above

5. In $4 - x = 4$, the _____ property applies.

(A) commutative
(B) associative
(C) identity
(D) none of the above

Complete the table. Use the table to answer questions 6-8.

quarters	1		3
nickels	5	10	

6. 10 nickels = _____ quarters.

(A) 2 (B) 5 (C) 3 (D) none of the above

7. 3 quarters = _____ nickels.

(A) 5 (B) 10 (C) 15 (D) none of the above

8. The ratio of nickels to quarters is _____.

(A) 1:5 (B) 5:1 (C) 5 − 1 (D) none of the above

9. Each soccer team has 11 players. What is the ratio of players to teams?

(A) 10:1
(B) 12:1
(C) 11:1
(D) none of the above

10. There are 9 soccer teams at the tournament. How many players are there?

(A) 20
(B) 2
(C) 100
(D) none of the above

Name _____ Date _____

Complete the tables. Use them to answer problems 21–23.

Table A

a	1			
$a+2$	3			

Table B

b	1			
$2b$	2			

21. In Table A, the values for $a+2$ are _____ numbers.

(A) always odd
(B) always even
(C) odd and even
(D) none of the above

22. In Table B, the values for $2b$ are _____ numbers.

(A) always odd
(B) always even
(C) odd and even
(D) none of the above

23. The values of a and b are located on the _____ of a coordinate graph.

(A) x-axis
(B) y-axis
(C) origin
(D) none of the above

---

24. _____, _____, _____, _____, _____, _____, _____, _____

(A) 3, –4, 4 (B) –4, –5, –6 (C) 3, 4, 5 (D) none of these

---

25. $\frac{20}{21}$, $\frac{18}{19}$, $\frac{16}{17}$, $\frac{14}{15}$, $\frac{12}{13}$, _____, _____, _____

(A) $\frac{11}{12}, \frac{10}{11}, \frac{9}{10}$ (B) $\frac{10}{11}, \frac{8}{9}, \frac{6}{7}$ (C) $\frac{13}{14}, \frac{15}{16}, \frac{17}{18}$ (D) none of these

---

26. .2, .4, .6, .8, 1.0, _____, _____

(A) 2.0, 3.0 (B) 10, 20 (C) 1.3, 1.5 (D) none of these

---

27. What is the value of x when $x^2 = 100$?

(A) 0
(B) 1
(C) 10
(D) none of the above

28. The distance between (1,6) and (–1,6) on a coordinate graph is _____ units.

(A) 12
(B) 6
(C) 0
(D) none of the above

Name _____ Date _____

9. In 5^3, the 5 is the _____.

(A) exponent
(B) order
(C) base number
(D) none of the above

10. An exponent is the number of times the base number is _____.

(A) multiplied
(B) subtracted
(C) added
(D) none of the above

11. Powers of 10, exponents with 10 as the base number, show _____.

(A) place value
(B) squares
(C) roots
(D) none of the above

12. The exponent with a base number of 10 is the _____ in the place value.

(A) number of 0s
(B) number of 1s
(C) number of 5s
(D) all of the above

13. The identity factor for subtraction is 2.

(A) True
(B) False

14. $n \div 3 = n$ is an example of the associative property.

(A) True
(B) False

15. $(7+3)+4=7+(3+4)$ is an example of the associative property.

(A) True
(B) False

16. Properties of operations are laws that apply to them.

(A) True
(B) False

17. In a list of different operations, perform the same operation from right to left.

(A) True
(B) False

18. The associative property uses order and parentheses to define equal expressions.

(A) True
(B) False

19. $20 \div 10 = (10 \div 10)+(10 \div 10)$ is an example of the distributive property.

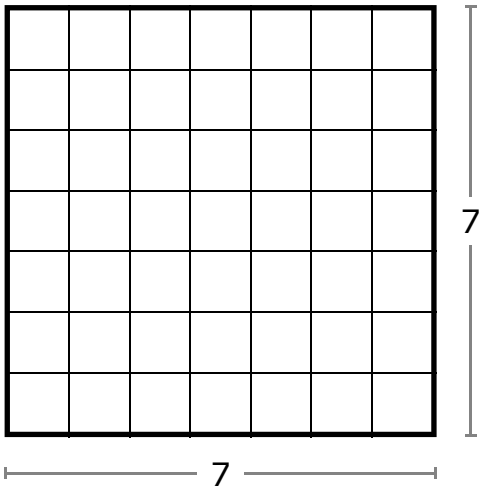
(A) True
(B) False

20. The identity factor for multiplication is 0.

(A) True
(B) False

Name _____ Date _____

Exponents

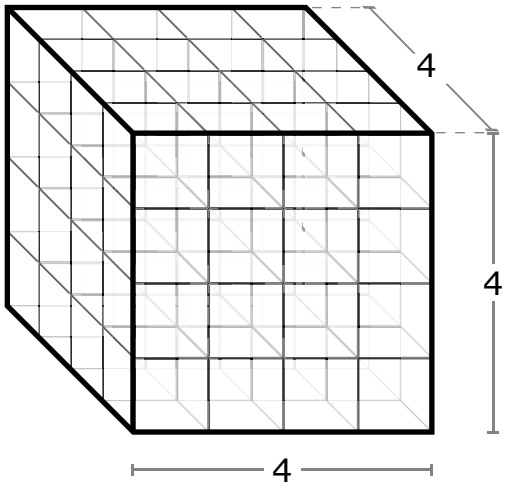


$7^2 = 7$ squared

7 is the base number.

2 is the exponent for the number of times the base number is multiplied.

$7 \times 7 =$ _____



$4^3 = 4$ cubed

_____ is the base number.

_____ is the exponent for the number of times the base number is multiplied.

$4 \times 4 \times 4 =$ _____

Name _____ Date _____

11. $9 + (3 \times 2) + (10 \div 5) =$ _____

(A) 15 (C) 17
(B) 16 (D) 18

---

12. In 5^4, the 4 is the _____.

(A) product (C) factor
(B) exponent (D) divisor

---

13. In 7^3, the 7 is the _____.

(A) product (C) difference
(B) sum (D) base number

---

14. $6^3 =$ _____

(A) 18 (C) 9
(B) 216 (D) 108

---

15. 4^2 is _____ 5^2.

(A) 9 less than (C) 2 less than
(B) 9 greater than (D) 2 greater than

---

16. $10^3 =$ _____

(A) 1,000 (C) 30
(B) 100 (D) 13

---

17. 3^4 is _____ 3×4.

(A) equal to (C) 69 less than
(B) 69 greater than (D) 24 greater than

---

18. $70{,}000 =$ _____

(A) 7×10^5 (C) 70×10
(B) 7×10^4 (D) 700×1

---

19. $10^2 \times 10^3 =$ _____

(A) 10^6 (C) 10^4
(B) 10^1 (D) 10^5

---

20. Exponents are used in writing _____.

(A) large numbers (C) both A and B
(B) scientific notation (D) none of the above

Name _____ Date _____

1. A cake recipe calls for 3 cups of flour and 2 cups of oil. Show the ratio of oil to flour. _____ : _____

   A. How many cups of oil are needed for 4 cakes? _____

   B. How many cakes can be made with 6 cups of flour? _____

---

2. A motorcycle travels 300 miles on 4 gallons of gasoline. A car travels 300 miles on 10 gallons of gasoline. Show both ratios.

   motorcycle = _____ : _____ car = _____ : _____

   A. How far does the motorcycle travel on 10 gallons of gasoline? _____

   B. How far does the car travel on 4 gallons of gasoline? _____

Name _____ Date _____

1. A ratio compares _____ quantities or amounts.

(A) 2
(B) 3
(C) 4
(D) none of the above

---

2. A ratio can be expressed as _____.

(A) 2:3
(B) 2 to 3
(C) $\frac{2}{3}$
(D) all of the above

---

3. A rate is a _____.

(A) race
(B) type of ratio
(C) rank
(D) none of the above

4.

Pencils	5	10	n
Pens	3	6	9

n =

(A) 11
(B) 12
(C) 13
(D) none of the above

5.

Weeks	1	2	3	n
Days	7		21	

n =

(A) 14
(B) 4
(C) 28
(D) none of the above

6.

Minutes			n	
Hours			3	

n =

(A) 60
(B) 120
(C) 180
(D) none of the above

7.

Quarters					n
Nickels					125

n =

(A) 24
(B) 25
(C) 26
(D) none of the above

8.

Bottles			3		n
Dollars			5		15

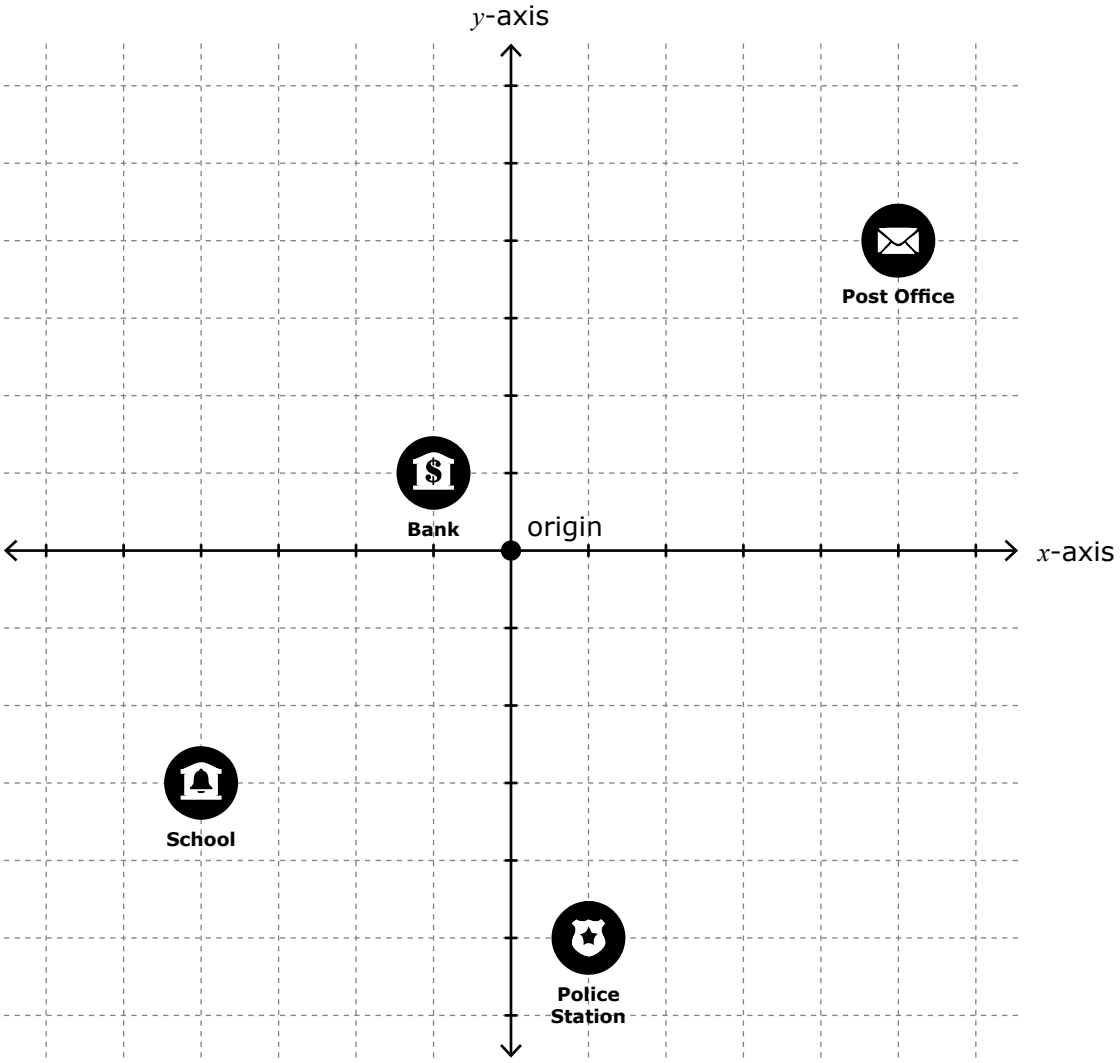
n =

(A) 10
(B) 5
(C) 4
(D) none of the above

Name _____ Date _____

City Map of Elizabeth

(Use with activity sheet 70)



1. Label each quadrant as a direction.
2. Label the x-axis as Main Street.
3. Label the y-axis as Central Avenue.
4. From the origin on the x-axis, the numbers to the right are east, The numbers to the left are west.
5. From the origin on the y-axis, the numbers at the top are north, the numbers at the bottom are south.
6. Each number is 1 block.

Name _____ Date _____

Write an example of how each expression might be used.

1. $\$5.75x$ _____

_____

2. $(640 - a)$ _____

_____

3. $1\frac{1}{2} + n$ _____

_____

4. $p \div \frac{1}{3}$ _____

_____

How are each pair of expressions different?

5. $84 - y$ and $y - 84$ _____

_____

_____

6. $t + t + t$ and $3t$ _____

_____

_____

Name _____ Date _____

Write an example of how each expression might be used.

1. $\$5.75x$ _____

_____

2. $(640 - a)$ _____

_____

3. $1\frac{1}{2} + n$ _____

_____

4. $p \div \frac{1}{3}$ _____

_____

How are each pair of expressions different?

5. $84 - y$ and $y - 84$ _____

_____

_____

6. $t + t + t$ and $3t$ _____

_____

_____

Name _____ Date _____

$4y+1$

When $y=1$, $4y+1=$ _____.

When $y=2$, $4y+1=$ _____.

When $y=3$, $4y+1=$ _____.

When $y=4$, $4y+1=$ _____.

$\frac{1}{2}c+5$

When $c=0$, $\frac{1}{2}c+5=$ _____.

When $c=2$, $\frac{1}{2}c+5=$ _____.

When $c=4$, $\frac{1}{2}c+5=$ _____.

When $c=6$, $\frac{1}{2}c+5=$ _____.

$7x-2$

For which value of x does $7x-2=12$?

(A) $x=0$
(B) $x=1$
(C) $x=2$

$30-3w$

For which value of w does $30-3w=15$?

(A) $w=0$
(B) $w=5$
(C) $w=10$

$.25p+1$

What value of x makes $.25p+1=2.25$?

(A) $p=3$
(B) $p=4$
(C) $p=5$

$2b-.5$

What value of b makes $2b-.5=11.5$?

(A) $b=2$
(B) $b=4$
(C) $b=6$

Name _____ Date _____

1.

$5 - $1.25 + $4 - $1.50 + $3 - _____ + _____

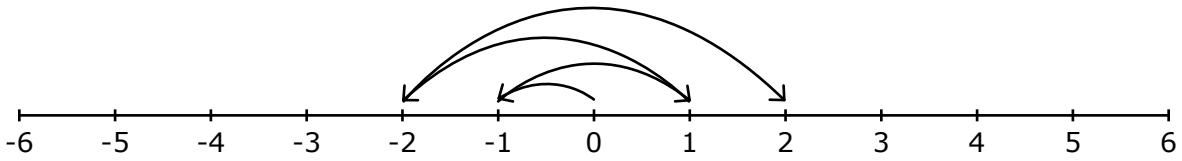
Describe the pattern using words.
I had $5. I spent $1.25. I earned $4. I bought some stamps for $1.50. I received $3 from my grandmother. I gave _____ to my brother for lunch money.

The pattern is _____

_____

_____

2.



Complete this pattern on the number line. Start at 0.
Describe the pattern on the number line. Use "right" and "left" in your description.

_____

_____

3.

$\frac{1}{5}$, 2, $\frac{2}{5}$, 4, $\frac{3}{5}$, _____, _____, _____, _____, _____

Describe the pattern.

_____

_____