# Learning Progression Workbooks from TOUCHMATH

## Division Sample Pages

Conceptually, subtraction is the foundation for division. Following are sample pages that demonstrate this progression.

**Multisensory Subtraction – Representational:**
Sample Page 1: From Red Workbook (book 3 of 6), Page 3

Students relate TouchPoints (mathematical manipulatives within our common numbers) with a ten frame model to boost an understanding of our place value system.

**Visual Models of Repeated Subtraction:**
Sample Page 2: From Aqua Workbook (book 5 of 6), Page 1

Students relate subtraction properties, make sense of patterns, and reinforce division as repeated subtraction. Use of number bond strategies and TouchPoints to help visualize the concept of a relationship of a number within base-ten (key pillar of elementary math) – developmentally appropriate way of learning division connected to subtraction as opposed to isolated as a new algorithm of division – boost conceptual understanding and meaning of division as a strategy for using larger numbers.

**Conceptual Understanding and Applying Models to Properties of Division:**
Sample Page 3: From Purple Workbook (book 6 of 6), Page 12

Students represent division with representational groupings of repeated subtraction, helping students move from Concrete (TouchPoints and manipulatives) to Representational (the numeral) to Abstract (solving problems using only mathematical reasoning).

**Division using Array Models:**
Sample Page 4: From Purple Workbook (book 6 of 6), Page 19

Students represent division using abstract division arrays to make sense of larger numbers, reinforcing place value, number sense, and divisive relationships – boosting conceptual understanding of the meaning of division.

The Division Learning Progression series contains six student workbooks leveled by color:

**Orange: TouchPoints & Comparing (K: CC)**
The foundation of the TouchMath Program and its multisensory approach to representing quantities and comparing numbers.

**Green: Understanding Subtraction (K: CC, OA)**
Understanding addition as putting together / adding to and understand subtraction as taking apart / taking from.

**Red: Multisensory Subtraction (K–1: OA, NBT)**
Developing subtraction strategies through multiple means of representation, action, expression, and engagement.

**Blue: Repeated Subtraction (1–2: OA, NBT)**
Building and representing fluency with subtraction and early division through repeated reasoning.

**Aqua: Understanding Division (2–3: OA, NBT)**
Developing an understanding of the meaning of division of whole numbers.

**Purple: Division (2–4: OA, NBT)**
Developing an understanding of operational relationships between division and subtraction.

The learning progression activities are carefully scaffolded. They may be used as a supplement to core instruction with Special Education students in a resource room setting, in a mainstreamed classroom, in a range of inclusion classrooms, and with remedial math students who need to be allowed to back up to the foundations. Additionally, progress monitoring is included to follow student progress.

**To learn more about Learning Progressions, or for more information on the Above+Beyond workbooks, visit www.touchmath.com/aboveandbyond or call 1-855-929-0880.**

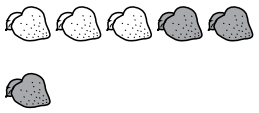
*Solve.*

There are 6 berries.

Olivia took **three away**.

How many berries are there now?

$$\begin{array}{r} 6 \\ -\ 3 \\ \hline \end{array}$$



_____ berries

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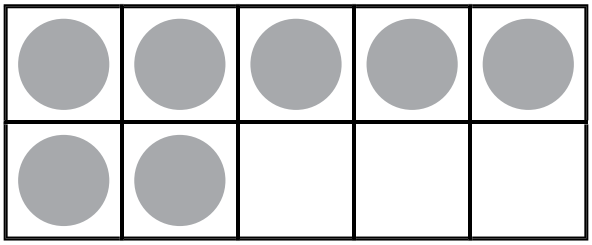
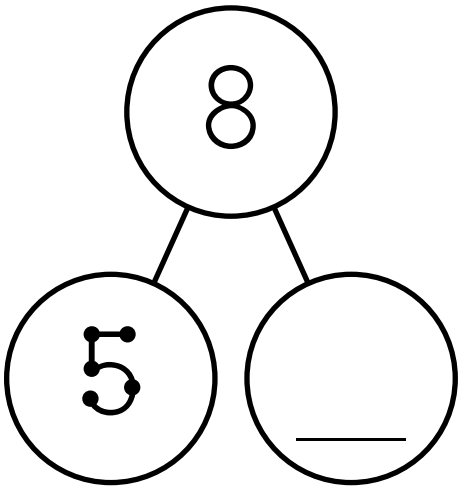
Draw an **X** on **two** dots below.



$$7 - 2 = \_\_\_\_$$

*Fill in the blanks.*




8 − 5 = _____

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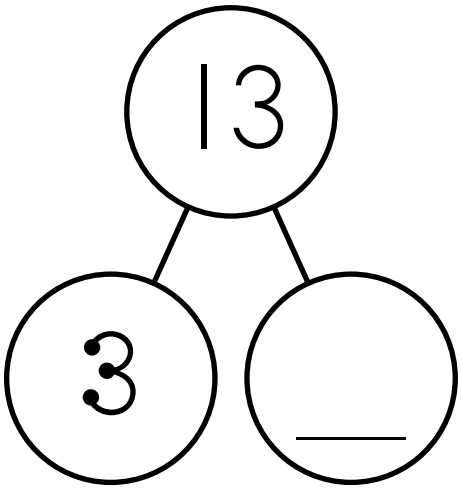


13 − 3 = _____

Write a subtraction sentence to find the unknown number for the number bond.

☐ − ☐ = _____

*Complete the problem below.*

$24 \div 8 = \_\_\_$

Use repeated subtraction to solve the quotient.

What multiplication sentence can help you solve the quotient?

*Draw an array and solve the word problem.*

18 circles are put into 9 rows.
How many circles are in each row?

Array:

27 circles are put into rows of 3.
How many columns are there?

Array: