# TouchMath® and NextGen Mathematics

Using manipulatives to boost every student's performance in the foundations of mathematics and critical thinking:

Evidence-Based

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By

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TOUCHMATH®

THE ALPHABET OF MATHEMATICS®

Since 1975

## Executive Summary

Current trends in employment indicate that how we work, with whom, from where and why will be different than what they are today. As educators, we are working to prepare students for taking their place in that world by providing them with the skills and knowledge to adapt to whatever they face and be successful. Organizations such as the World Economic Forum, Gartner, Forbes and Brookings state that the skills and knowledge that will be needed by our students as they enter careers and become citizens are creativity, critical thinking, decision making, and complex information processing. (George, 2017; Henry-Nickle, 2018; Jezard, 2018; & Tunkel, 2018.)

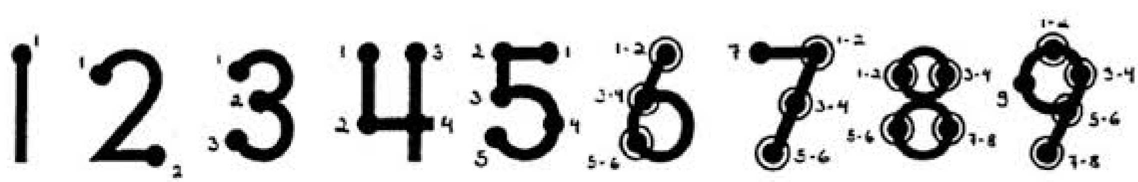
Ubiquitous to these are the literacies needed to function in the world-digital, mathematical, civics, foundational, etc. Those who understand and can demonstrate these skills will have more opportunities and options in their futures. It is essential that everyone understand mathematics and fortunately, we live in an age where we know that all students can learn mathematics with the right supports. In order to develop mathematical literacy-translating mathematical concepts and knowledge into plain-language, one must understand mathematical concepts, and while it is doable for all students, it is often challenging for students with disabilities, gaps in their schooling or poor language skills. (NCTM, 2015)

Historically, educators have used concrete manipulatives to support and build conceptual understanding. TouchMath® Counting is the most basic math skill and connected to every mathematical domain area for elementary and middle school. TouchMath®is the only curriculum program that uses a numeral as a manipulative, instantly making a potentially daunting abstract concept into real and concrete action. It is also a multi-sensory curriculum that uses seeing, hearing, and touching numerals. This multi-sensory approach allows students to engage in mathematics visually, tactically, and kinesthetically.

Research supports the effectiveness of TouchMath® as positively impacting students performance in cardinality, operational algorithms, conceptual understanding, application, fluency, automaticity, and modeling. (Kot, Terzioglu, & Yikmus, 2018) “Good mathematics teachers typically use visuals, manipulatives, and motion to enhance students' understanding of math concepts." (Boaler, 2018) The importance of visualization and multiple representations of math concepts can be supported by the work of both the National Council for the Teaching of Mathematics (NCTM) and the Mathematical Association of America (MAA) which have long advocated for the use of multi-sensory approaches in mathematics to promote engagement and conceptual understanding.

TouchMath® has the ability to bridge the gap between concrete and representational mathematics. The program takes into account students' academic and cognitive proficiencies and deficiencies while enriching both the concept and the computation. The sequences that TouchMath® follows are built upon the work of Dr. Jean Piaget and Dr. Lev Vygotsky. The work of both Piaget and Vygotsky led to some of the most utilized constructivist methodologies and developmental theories of our modern educational systems. (Bullock, 2009)

The multi-sensory approach of using touchpoints to an abstract item such as a number, helps students conceptualize the total quantity of digits.
When students conceptualize quantities without degradation of their working memory or executive processes, students gain procedural and conceptual content at an increased rate. Cognitive systems and neuroscience research suggests ". . . that the neurobiological basis of mathematics cognition involves complicated and dynamic communication between the brain systems for memory, control and detection and the visual processing regions of the brain." (Boaler, 2018)



*Example of TouchMath® Touch Points*

Eventually, students progress their mathematical knowledge of the number of dots, or touchpoints, on a digit that corresponds with the quantity of a number. This progression from concrete to pre-representation allows students to gain access to deeper levels of mathematical understanding. TouchMath® has a growing body of independent, primary and secondary research that continues to validate the fundamental principles of the TouchMath® system and point to a common conclusion: TouchMath®'s multisensory approach and sequential strategy improve students math comprehension and mastery. TouchMath® manipulatives and visualization strategies have been used in classrooms throughout the world to help all students progress from concrete through representational to abstract mathematical proficiency. Research in TouchMath® as a program compared to other math interventions showed that students ". . . solved math problems faster and more accurately than their counterparts." (Dulgarian, 2012)

## Background

The purpose of this whitepaper is to demonstrate clearly and explicitly the scientific research base supporting the use of the TouchMath® program's use of visuals and manipulatives to help students performing at, above, and below grade level to progress through the CRA-continuum. This includes students needing specific remediation, English Language Learners, Students with Disabilities, RTI-intervention, and Next Generation Math Standards support. TouchMath® has gained support throughout the 43 years of practical use with teachers and intervention specialists having successfully implemented the TouchMath® program in a multitude of settings. (Heller, 2011) (Yikmus, 2013) (Sonmez & Cora-Ince, 2016) (Yikmis, 2016) (Kot, Terzioglu, & Yikmus, 2018)

TouchMath® was initially developed in 1975 by Janet Bullock. She found that many students were struggling with math concepts and were in need of an intervention that would increase their math skills as well as their confidence in the subject. The TouchMath® program is firmly established in the works of Jean Piaget and Jerome Bruner. (Bullock, 2009) The learning concept of moving through a framework of concept stages: Concrete, pictorial, and symbolic was the early basis of the TouchMath® program. TouchMath® aids in students procedural fluency, one-to-one correspondence, stable order, cardinality, abstraction, order irrelevance, and conceptual development. TouchMath® is a researched based program. The below graph contains TouchMath® research that was completed over the past twenty-five years and proves the effectiveness of the TouchMath® program.

| Source | Dependent Variable | Independent Variable | Participants | | Research | Finding |
|---|---|---|---|---|---|---|
| | | | Number | Diagnosis | | |
| Newman, 1994 | Basic addition operation | TM | 4 | DS | MPDAP | Effective |
| Simon ve Hanrahan, 2004 | Addition operation | TM | 3 | LD | MPDAP | Effective |
| Çalık, 2008 | Addition operation | TM | 3 | MR | MPDAP | Effective |
| Cihak ve Foust, 2008 | Addition operation | TM & NL | 3 | ASD | ATM | TM is more effective and efficient |
| Fletcher, Boon ve Cihak, 2010 | Addition operation | TM & NL | 4 | MR-ASD | ATM | TM is more effective and efficient |
| Avant ve Heller, 2011 | Basic addition operation | TM | 3 | PD | MPDAP | Effective |
| Eliçin, Dağseven- Emecen ve Yıkmış, 2013 | Addition operation | TM | 3 | MR | MPDAP | Effective |
| Kot, Sönmez, Yıkmış ve Cora-İnce, 2016 | Addition operation | TM | 3 | HD | MPDAP | Effective |
| Yıkmış, 2016 | Addition operation | TM | 3 | ASD | MPDAP | Effective |
| Kot, Sönmez ve Yıkmış, 2017 | Basic addition operation | TM & NL | 2 | MR | ATM | TM is more effective and efficient |
| **Research Involving Addition And Subtraction** | | | | | | |
| Scott, 1993 | Addition and Subtraction | TM | 3 | MR | MPDAB | Effective |

TM: Touch Math, NL: Number Line, MR: Mental Retardation, DS: Down Syndrome, LD: Learning Disability, ASD: Autism Spectrum Disorder, PD: Physical Disability, HD: Hearing Disability, MPDAB: Multiple Probe Designs Across Behaviors, MPDAP: Multiple Probe Designs Across Participants, ATM: Alternating Treatments Model.

*Diagram 1: (Kot, Terzioglu, & Yikmus, 2018)*

The following provides evidence for how TouchMath® supports the Next Generation Mathematics Standards, Universal Design for Learning, Standards of Mathematical Practice, Response-to-Intervention, Research-Based Best Practices, Mathematical Progressions, Visualizations in Mathematics, CRA-Continuum, and the Mathematical Mindset.

**Using manipulatives and visuals to boost every student’s performance in the categories of mathematical proficiency.**

The notion that mathematics is only accessible to students who are quick with number facts, can instantly quantify objects, or who are just plain “math people” is false. With new research in neuroscience, we have identified the importance of the skills that underlie mathematical proficiency. The “categories of mathematical proficiency” are interwoven with neuroscience research and research-based best practices.

- The first category of mathematical proficiency is **conceptual understanding**, which is the comprehension of mathematical concepts, operations, and relations. TouchMath® boosts conceptual understanding through the reinforcement of visual manipulatives. Students access multiple levels of conceptual understanding throughout a standard Common Core-aligned math lesson. The goal for students is to access deeper meanings of mathematical operations, relationships, and concepts. The problem in a standard classroom is that students have difficulty accessing the "comprehension" of the concepts without a firm root in the visualization and modeling with the concepts. (Boaler, 2018)

- **Procedural fluency** is typically defined as the skill in carrying out procedures flexibly, accurately, and appropriately. (National Research Council, 2001) When students struggle with procedural fluency, they cannot "apply" math. This manifests in many forms, but commonly as a deficiency in procedural fluency that will lead to students saying, "I need help, I don't know how to start this problem." TouchMath® can help resolve procedural fluency deficits by reinforcing the CRA-continuum. The act of visualization in TouchMath® is based in manipulative strategies, which increases flexibility, accuracy, and cohesion.

- The third category of mathematical proficiency is **strategic competence**. This manifests as a student's ability to formulate, represent, and solve mathematical problems. If a student struggles with strategic competence, they may try to solve a word problem, struggle with the operation and representation component, and then end up drawing 1,000 tally marks. Without a proper systematic approach, such as TouchMath®, students who lack strategic competence are unable to represent the mathematics, thus lacking any justification for their answers. Strategic competence is closely related to the Common Core State Standards (CCSS) and Standards of Mathematical Practices (SMP). (National Research Council, 2001)

- **Adaptive reasoning** is a student's capacity for logical thought, reflection, explanation, and justification. Successful math students must think logically about relationships between concepts and situations. This strand closely relates to the Standards of Mathematical Practices 3, which states "Mathematically proficient students reason abstractly and quantitatively." Students must make sense of quantities and their relationships when problem-solving. This is an important math skill for students to possess and involves creating coherent arguments and using symbols to represent mathematical situations. The TouchMath® Program results in students who are able to use the different properties of operations and objects flexibly, thus bringing adaptive reasoning to the next level in any type of classroom.

- **Productive disposition** is the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. Making sense of problems and persevering in solving them is a strong skill to have. Students in the TouchMath® program are trained to explain the meaning of a problem and actively look for ways that it can be solved, which is derived from the manipulative strategies of counting on the digit. Instead of jumping right into an attempt at a solution, students need to critically analyze the math problem, speculate about the form and meaning of the solution, and plan a pathway to get there. The TouchMath® strategies help guide students in procedural steps that increase accuracy and focus. Children that are successful in mathematics see mathematics as meaningful, interesting, and accessible. (Illustrative Mathematics, 2014)

**Using manipulatives and visuals to boost every student's performance in Next Generation Math Standards.**

The TouchMath® Program has been optimized and links key areas of the Next Generation Math Standards to improve student performance. The attributes of the standards include:

1) Greater **focus on fewer topics**
2) **Coherence**: Linking topics and thinking across grades
3) **Rigor**: Pursue *conceptual understanding*, *procedural skills* and *fluency*, and *application* with equal intensity

The developmentally appropriate practices are a framework of principles that promote children's learning and development. The link between early childhood mathematics education and the Next Generation Math Standards must take into account the developmental framework of the child's age. The TouchMath® Program elevates classroom instruction by providing coherence between kinesthetic learning and the overarching goals within the Next

Generation Math Standards to focus on deeper conceptual knowledge and procedural fluency.

TouchMath® allows students to demonstrate knowledge of mathematics in a developmentally appropriate way. Young children need a multi-sensory approach to mathematics. When students use the touch points in TouchMath®, they are exploring the interrelated nature of developmental domains where learning in one area will influence another. The connection of developmentally appropriate domains with topics and standards is called cohesion, which has been proven to positively benefit student performance, mathematical comprehension, retention, and acquisition of new materials. TouchMath® is embedded in a wealth of research, revealing both program efficacy and academic impact for all students.

Since TouchMath® is used as a pre-representational multisensory technique, prior experiences that may shape a child's behavior and approaches to learning are stabilized, allowing the child access to the root of number sense in a fun and engaging way. With the systematic approach to differentiated and tiered instruction, TouchMath® has a positive impact on all students, in all settings.

**Using manipulatives and visuals to boost every students performance in the Universal Design for Learning.**

The Universal Design for Learning plays a pivotal role in both the TouchMath® Program and student achievement for a wide range of learners: general education, special education, intervention, remediation, English Language Learners (ELL's), and students performing above or below grade level. In order for students in the classroom to become active, motivated, and successful learners, a deeper level of instructional context must take place. The Universal Design for Learning is derived from research-based best practices in cognitive neuroscience and takes into account the diversity of learners physiologically, psychologically, and socio-emotionally. "Neuroscience reveals tremendous differences in how individuals, learn even among those who on the surface seem to have a lot in common...there is a distinct variability not just from person to person, but from *within* individuals." (Rose, 2012)

The Universal Design for Learning is based on three main principles of learning: representation, action & expression, and engagement. (CAST, 2018) These three learning principles are connected to three corresponding neural networks, which include respectively, the recognition, strategic, and effective neurological networks of learning. (CAST, 2018) TouchMath® is deeply grounded in scientifically based practices that correspond to the Universal Design for Learning framework. In practical applications, UDL and TouchMath® complement one another.

The Universal Design for Learning is a framework for developing learners who are resourceful and knowledgeable by providing **multiple means of representation**. Representation activates the recognition network of the brain, which is the "what" of learning. The TouchMath® program offers ways of customizing the display of information, allowing students to benefit from the perception of mathematics, thus maximizing transfer and generalization of key math concepts. The language and symbols used in the TouchMath® system support the decoding of text, mathematical notions, and symbols. Students gain cohesion within the math content strands by activating background knowledge back to the manipulative of a digit during this representation frame. Teachers are able to highlight patterns, critical features, big ideas, and relationships through the use of a proven method of instruction through touch point for all operations, linking to the very fabric of the Common Core State Standards and Response-to-Intervention frameworks.

The multiple means of **action and expression** embedded within the TouchMath® program develop learners who are strategic and goal oriented. Action and expression activate the strategic network of the brain, which is the "how" of learning. The TouchMath® program promotes varying methods for response and navigation, which at its core are multi-sensory and grounded in scientifically proven methods for intervention. Multiple means of expression promote beneficial mathematical discourse between teacher-to-student and even student-to-student interactions. This type of mathematical discourse promotes fluency and offers graduated levels of support for practice and performance. The "strategy" of applying touch points to numbers while accessing higher level math problems helps students with Executive functions and also supports both planning and strategic development.

The goal of the TouchMath® Program is to develop learners who are purposeful and motivated by providing **multiple means of engagement**. Engagement links to the affective networks of neurology, which can be defined as the "why" of learning. Recruiting interest by minimizing distraction, sustaining effort and persistence by varying demands, and the development of strategies by developing self-assessment are the pillars of strategies behind the TouchMath® program.

With targeted intervention for students performing above grade level, below grade level, students with disabilities, and English language learners, TouchMath® offers a multi-tiered program to fit the needs of any classroom or curriculum.

**Using manipulatives and visuals to boost every student's performance in the Standards of Mathematical Practice.**

The Standards of Mathematical Practice are a series of practices that strong mathematics educators use at all levels and should seek within their students. These standards focus on the key "processes and proficiencies" successful math students exhibit when working through complex problems, communicating results, and accessing conceptual levels of understanding in key domain areas. It is paramount that we recognize opportunities to emphasize the Standards of Mathematical Practice during activities that promote conceptual understanding.  Conceptual understanding in mathematics can take the form of:

- Concrete and pictorial models
- Real-world contexts
- Conceptual questioning
- Speaking and writing about understanding

Within the best-practices, cross-grade coherence is accessible through conceptual understanding which improves students' ability to not only learn math concepts but to have those concepts stick along the learning trajectory. (Illustrative Mathematics, 2014) The eight Standards of Mathematical Practice are closely linked to the TouchMath® Program, as represented below:

**(1) Make sense of problems and persevere in solving them.**

The TouchMath® Program has embedded strategies that enable students to access and perseverance by using touch points for operations (addition, subtraction, division, and multiplication) to alleviate strain on working memory and keep students focused on the content task at hand. (Dulgarian, 2012)

**(2) Reason abstractly and quantitatively.**

With TouchMath®, the visual strategies bridge the gap between concrete manipulatives and representation of deeper levels of math domain areas. The result is increased proficiency in both quantitative knowledge and abstract reasoning. (Kot, Terzioglu, & Yikmus, 2018)

**(3) Construct viable arguments and critique the reasoning of others.**

Students often struggle with the ability to not only create mathematical justifications during application problems, but they also have difficulty critiquing the reasoning of their peers. With metacognition being a key pillar for student retention of mathematical knowledge, many students miss out on mathematical discourse. TouchMath® can assist students in organizing their mathematical thinking to promote math discourse in the classroom by building operational automaticity with gradual levels of support and differentiation.

**(4) Model with mathematics.**

TouchMath® combines researched-based CRA-continuum strategies with Common Core-aligned modeling. The outcome, successful students who can navigate the conceptual levels of mathematics through multiple means of representation, a key pillar of any successful math student.

**(5) Use appropriate tools strategically.**

Since TouchMath® uses numerals as concrete manipulatives through touch points, students have a key tool at their disposal whenever doing mathematics. This can also aid in the proper use of strategic tools such as rulers, calculators, and protractors.

**(6) Attend to precision.**

TouchMath® increases students ability to sustain effort and positively influences their perseverance throughout math problems. Precision and perseverance are key pillars of the Common Core State Standards and the Next Generation Common Core Math Standards.

**(7) Look for and make use of the structure.**

Mathematically proficient students look closely to discern a pattern or structure. (Illustrative Mathematics, 2014) When students identify and make use of structure, they have not only a higher probability of success but have a greater chance to access higher levels of conceptual understanding. The use and structure of patterns within the TouchMath® numerals assists students in seeking a deeper understanding of the "structure" of the problem as opposed to finding the answer without the justification of "why."

**(8) Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. TouchMath® is a multisensory math program, and at its essence trains students to look for repeated reasoning. The program helps make math concepts easier and more accessible for students with different learning styles or learning difficulties. The approach uses auditory, visual and tactile strategies for understanding numbers and operations.

The TouchMath® Program allows students the accessibility of conceptual understanding within deeper levels of mathematical complexities. "Teachers teach more than 'how to get the answer' and instead support students' ability to access concepts from a number of perspectives, so that students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate a deep conceptual understanding of core math concepts by applying them to new situations as well as writing and speaking about their understanding." TouchMath® allows students to bridge gaps in prerequisite knowledge necessary to activate conceptual understanding and mathematical discourse.

**Using manipulatives and visuals to boost every student's performance in Response to Intervention.**

Response to Intervention (RtI) is an early detection, prevention, and support system that attempts to identify and assist struggling students with appropriate levels of intervention. The essential components for implementing a successful RtI framework includes high-quality, scientifically based classroom instruction, ongoing student assessment, and Multi-Tiered instruction.

**Tier 1:** Students learn at roughly grade level or above and are least likely to fall behind or need intervention.

**Tier 2:** Students lag well behind their peers, demonstrate weak progress on screening and measures, and require some form of intervention.

**Tier 3:** Students lag behind their peers by one or more years, demonstrate very weak progress on screening measures, and require intensive intervention.

The TouchMath® Program provides both a support system and multi-level intervention system to assist students who fall within the Tier 1, 2 and 3 levels of the federally mandated Response-to-Intervention framework. The key components of the RtI framework include: screening, data-based decision making, progress monitoring, and a multi-level prevention system.

Researchers advocate that students with learning disabilities, particularly in math, receive engagement in learning with an application, plenty of feedback, and teaching that correlates with personal learning preferences. TouchMath® provides these strategies with engagement, feedback, and multisensory methods. The study shows (Mays, 2008):

- Use of the TouchMath® strategy increases computation skills and additionally promotes a computation strategy that students use independently.
- Seventeen of 34 students met the goal of 100% improvement on computation tasks.
- Decreasing errors as students learn the TouchMath® strategy and continued use of the program improves student motivation and test scores.
- Knowledge of TouchPoints® allows ease of use and faster-timed scores.

TouchMath® has been proven to be an effective RtI intervention tool that will assist in helping students access math curriculum.

**Using manipulatives and visuals to boost every student's performance in the best research based best practices.**

**Procedural skill and fluency**: Mathematically proficient students are expected to have speed and accuracy with simple calculations; teachers structure class time and/or homework time for students to memorize through repetition, core functions." In order to bridge the gaps of prerequisite knowledge, students must gain numerical fluency. TouchMath(r)'s research-based program has been shown to positively improve student performances in computational and procedural fluency. (Kot, Terzioglu, & Yikmus, 2018)

**Application:** Students' ability to use relevant conceptual understandings and appropriate strategies and tools even when not prompted to do so. "Students are expected to use math and choose the appropriate concept for application even when they are not prompted to do so. Teachers provide opportunities at all grade levels for students to apply math concepts in 'real world' situations. Teachers in content areas outside of math, particularly science, ensure that students are using math – at all grade levels – to make meaning of and access content." (excerpt from the Shifts) Application within the Next Generation Math Standards can be applied in three variations: word problems, real-world problems, and mathematical modeling. Mathematical modeling is very important for students ranging from Pre-Kindergarten to Grade 12. Modeling within the Next Generation Math Standards and the

Common Core State Standards can be broken into descriptive modeling and analytic modeling.

Descriptive modeling is using concrete materials or pictorial displays to study quantitative relationships. Analytic modeling uses graphical representations, equations, or statistical representations to provide analysis, revealing additional insights into the relationships between variables. The key difference lies in whether the modeling simply facilitates informative visualization or allows for new and deeper analysis.
(CCLS for Mathematics, p55-56)

Descriptive modeling in Pre-Kindergarten to Kindergarten allows students to model their environment and real-world objects using geometric shapes and figures. This progresses from Kindergarten to first grade, where students developmentally start to form models of addition and subtraction situations using concrete materials or diagrams. In grades two to five, students access descriptive modeling through place value models, multiplicative models (equal groups, array, area), visual fraction models, and number line models. Within the upper grades of grades sixth through twelfth, descriptive modeling takes the form of tables or graphs of observations, geometrical, and/or 3-D models of objection.

Analytic modeling is accessed by grades four through seven by the use of tape diagrams and/or equations to model relationships between two or more quantities. Analytic modeling middle and high school math use equations and/or functions to model relationships between two or more variables.

**Cohesion:** Cohesion is the concept within best-practices to have math domain areas work together seamlessly. The TouchMath® program ties in both research-based best practices and the structure of learning trajectories to help students build conceptual understanding through cohesion points throughout the curriculum. Each of these areas of application can be used to cohesively bridge gaps in prerequisite knowledge. The Standards of Mathematical Practice can also help promote application for mathematics. Within the TouchMath® Program, word problems and the strategic approach to operational relationships take precedence over ROTE isolation of facts. Real-world, problem-based questions allow students to become engaged in meaningful, accessible, and conceptually-rich mathematics; however, without the proper scaffolding, and the aid of a proven multi-sensory program such as TouchMath®, a large percentage of students will not be able to access the mathematics at this level.

**Using manipulatives and visuals to boost every student's performance in the progressions of mathematics.**

The Mathematical Progressions between the Common Core State Standards focuses on the progression of a topic across a number of grade levels, embedded in educational research, research-based best practices, and the structure of mathematics. We would be remiss to discuss standards and instruction without focusing on the "glue" that holds mathematics together. This narrative, of progressing structure, relationships, and content assists in helping both teachers and students maintain conceptual understanding and the ability to build off prior knowledge without jumping from concept to concept in a meaningless way.

Just like great literature has a story, thread, a plot sequence, relationships with characters, mathematics at its core is interconnected. TouchMath® can assist the flow of the mathematical progressions that follow a standard from the start of the content "strand" to the end of the domain area. This effect is termed a "learning trajectory." A learning trajectory is closely related to the domain areas in the Common Core State Standards for Mathematics. In elementary math instruction, the domain areas include counting and cardinality, operations and algebraic thinking, numbers and operations in base ten, geometry, measurement and data, and fraction operations. The TouchMath® program can assist teachers in implementing, with fidelity, the rigor of the CCSS and help shorten the deficits of students performing below-grade level or students struggling to gain concepts.

The Common Core Standards for Mathematics ask students to develop both conceptual and procedural knowledge, as well as engage in mathematical practices such as perseverance, pattern identification, and making sense of structure. (Lambert, 2018) Conceptual and procedural knowledge within the CCSS is held together by both the mathematical progressions, learning trajectories, and mathematical modeling.

A prime example of the shift from the learning spiral to the mathematical progressions (learning trajectory) approach can be seen through the lens of a 5th-grade math student that is struggling with fractions. At its core, a student struggling with fractions may appear to have difficulty understanding the basic structure of a fraction; the divisive relationship between the numerator and denominator, application of operations to the fraction, or even what the fraction represents.

Without the progressions and the framework of Common Core, a teacher may continue to reinforce the fractional procedures and concepts until the student seems to "master" the skill, only to realize they "forget" the procedure after the unit has passed. This is all too common. However, if the teacher were to use the mathematical progressions, and take into consideration a learning trajectory, the focus would be on the domain of "numbers and operations - fractions."

If a student is struggling with the fifth grade standard of 5.NF.1: *Use equivalent fractions as a strategy to add and subtract fractions,* then they can identify coherence to the 4.NF standard. These standards of equivalence of fractions in both grades 5 and 4 are inter-connected to the grades 3 and 2 multiplication and division strands, which can be interpreted in this case as "representing basic fractional relationships on a number line or using another visual modeling." This domain strand further moves down to grades 1 and 2 for repeated addition and subtraction in the operations and algebraic thinking strand. The most important foundation, based on the progressions of mathematical concepts for K-5 fractional understanding, is geometry. The composition and decomposition of shapes actually is early division and fractional understanding.

In order to understand fractions in fifth grade, Kindergarten teachers can start focusing on geometric shapes. This relationship is a prime example of a progressional learning trajectory that is connected both mathematically at its core and grounded in research-based best practices of curriculum sequencing. TouchMath® has the Progressions of the CCSS built into its core, allowing students to seamlessly move through the learning trajectories of both the Next Generation Math Standards and the Common Core State Standards.

TouchMath® offers support in Operations and Algebraic Thinking, Geometry, Counting and Cardinality, Numbers in Base Ten, Fractions, Measurement and Data, and Ratios. TouchMath® uses visual cues to help students learn in a multi-sensory way. TouchMath® is carefully designed to be taught in a sequential and developmentally appropriate way. The strategies used in earlier TouchMath® help students access higher levels of math. The visual cues assist students can generalize conceptual knowledge towards other domain areas within their foundational and grade-level standards.

**Using manipulatives and visuals to boost every student's performance in visualizations and mathematics.**

The importance of visualization representation in mathematics is scientifically backed in classroom best-practices, but neuroscience is just beginning to understand the profound ways our minds learn. Scaffolding, differentiated instruction, and representing information through multiple means assists students to both internalize and generalize information. Students learn best when information is multi-sensory, accessible, and conceptually engaging.

Visualization in the math classroom is supported by the Next Generation Math Standards (Common Core State Standards), Mathematical Progressions, Universal Design for Learning, CRA-Continuum, and neuroscience. Chunking information together in a visual modality helps students summarize smaller pieces of minute details and increases the connection to conceptual connections, such as learning trajectories, mathematical domains, and mathematical relationships (operational, conceptual, procedural, and algorithmic). Examples of visualization in mathematics include ten-frames, number bonds, tape diagrams, array and area models, ratio tables, and double number lines. Visualizations in mathematics assist students in unlocking deeper levels of conceptual understanding and aid in relieving some of the processing students are using at the moment to think deeper about concepts rather than procedures. This in effect allows students to go deeper into the math, or "an inch wide and a mile deep" as opposed to the surface knowledge of "a mile wide and an inch deep" which is often used as an analogy to pre-common core mathematics (lacking both depth of knowledge and the interconnectedness of the common core progressions).

TouchMath® utilizes visualization strategies in mathematics. TouchMath® is a multisensory math program that makes critical math concepts appealing and accessible for students who struggle to understand grade-level content. TouchMath® firmly reinforces the Common Core State Standards, with an emphasis on a blended solution of print and technology.

**Using manipulatives and visuals to boost every student's performance in the CRA-continuum.**

The TouchMath® program is grounded within the CRA-Continuum. The Concrete-Representational-Abstract Continuum is a sequence of instructional practices and research-based best practices that move from an instructional focus on concrete representations (manipulative materials) and models to semi-concrete representations (drawings or pictures) and images to abstract (using only numerals or mentally solving problems). (Van de Walle 2012, 8th Edition)

The concrete technique for mathematics instruction helps learners interpret ambiguous or opaque abstract symbols in terms of well-understood concrete objects. Concrete instructional practices also aid in providing perceptual and physical experiences that can ground abstract thinking, enabling learners to build up a store of memorable images that can be used when abstract symbols lose meaning, and guiding learners to strip away extraneous concrete properties and distill the generic, generalizable properties. (Fyfe 2014) The TouchPoints® within the TouchMath® program is a pre-representational stage and a bridge to help guide students away from standard manipulatives and put their attention back into the math. The TouchMath® multi-sensory approach is a highly effective means of having students progress passed the concrete stage of learning.

**Using manipulatives and visuals to boost every student's performance in math literacy.**

Math literacy is a key shift in the way teachers facilitate math learning. Mathematics, unlike the English Language Arts, is taught in a performance-based way. TouchMath® is a game-changer for teachers looking to implement both intervention strategies and support students access to the Common Core State Standards. Often math is misrepresented as a series of rules, isolated parts of a puzzle that only specialists get to unlock. Most of us remember the feeling when the teacher calls on you to demonstrate a math problem. Positive or negative connotation? Even as teachers, we shy away from showing others our work in math, why? This ingrained belief that math is about performance, pressure, right vs. wrong is entrenched in our systems of teaching, learning, and curricula. At its very core, math is not about right or wrong, it is not dualistic, but a thread that interconnects disciplines (science, technology, engineering, social studies, ELA), concepts, and relationships (operational and contextual). A strong math teacher will always relate mathematics to a puzzle, a real-life connection can emerge into just about every action we do in our lives.

If students see the link between open-play and functionality, they will have a stronger motivation to learn. A negative belief system regarding their performance in math can have a dramatic effect on their actual performance even if the issue is not learning oriented. One example is the age-old statement "I am not a math person." This negative self-belief can impact students confidence in their abilities as a math student, stripping away the very fabric of the fun, engaging, and conceptually rich content area of math. TouchMath®'s multimodality approach using representations and manipulatives in the forms of everyday numbers, aids in even higher levels of math concepts, particularly with modeling and representation. There are ways to help support students' positive beliefs about math performance and help foster a mathematical growth mindset:

- Praise effort over outcomes.
- Encourage students to embrace challenges.
- Give students time to engage in deep thinking and conceptual thought.
- Celebrate mistakes as learning opportunities.
- Assist students with positive beliefs about themselves.
- Productive struggle through perseverance. (Dweck, 2008)

## Conclusion

With a foundation in research-based intervention practices, TouchMath® offers a strong approach to differentiating instruction through the Universal Design for Learning (UDL) Framework. Research-based Universal Design for Learning has provided a structure for thinking about how to meet the needs of our diverse classrooms using strategies grounded in scientific research and best practices. With numerals as manipulatives, personalizing learning and enrich entry points for learning standards, TouchMath® can be the game-changer for students learning math. A typical math classroom in America can have a wide range of students within: students performing above grade level, students performing below grade level, students with disabilities, and English language learners.

TouchMath®'s multi-sensory approach allows students of all proficiency levels access to multiple layers of concepts within the fabric of math. TouchMath® is researched-based and proven to be effective in multiple classroom settings: general education, special education 12:1:1, Co-Integrated settings, Inclusive and Mainstreamed classrooms, English language learner classes, Academic Intervention Services, and self-contained classrooms. The goal of any classroom is to be least restrictive for the students. TouchMath® provides a variety of learners with the experiences, strategies, and support needed to access the key domain areas of the Next Generation Common Core Math Standards (CCSS). The common thread between general education, special education, and academic intervention is the need for students to develop a strong conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. (National Research Council, 2001)

## About the Authors

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